2015
The Graduate School Entrance Examination
Physics
9:00 am－11:00 am

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Print your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of the answer sheet represent the problem number you answer (P 1, P 2, P 3, P 4) on that sheet and also the class of the master's course (M) and doctoral course (D) applicants. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks on each sheet with a pair of scissors.

6. You may use the blank sheets of the problem booklet as working space and for draft solutions, but you must not detach them.

7. Any answer sheet with marks or symbols irrelevant to your answers will be considered invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

Consider a conical surface with a half apex angle of 45° and a bottom radius $2R_0$. As shown in Figure 1.1, the conical surface is placed with the vertex down and with its axis vertical. The vertex has a small hole. There is a string placed through the hole with mass points 1 and 2 (mass m) attached at both ends. Diameter of the hole and thickness of the string are sufficiently small. Mass and stretch of the string and friction are negligible. The acceleration of gravity is denoted by g. Answer the following questions.

I. Mass point 1 is in horizontal uniform circular motion on the conical surface with a radius R_0 and a velocity v_0. Obtain the velocity v_0 of mass point 1.

II. The system is in motion stated in Question I. When the string breaks suddenly, mass point 1 moves upward on the conical surface.

1. Obtain the angular velocity of mass point 1 about the axis of the conical surface and the vertical component of the velocity of mass point 1 at the moment when mass point 1 reaches a height H from the vertex. Use v_0 in both equations.
2. Answer with reason whether mass point 1 flies out of the top edge of the conical surface or not.

III. The motion of mass point 2, stated in Question I, was vertically perturbed. Then, mass point 2 started vertical small-amplitude oscillation.

1. Let the position of mass point 1 be expressed in cylindrical coordinate (h, r, θ). The origin of the coordinate system is the vertex of the conical surface, and the h-axis directs vertically upward. The following differential equation describes the motion of mass point 1. Obtain the coefficients a and b.

$$\frac{d^2r}{dt^2} + ar\left(\frac{d\theta}{dt}\right)^2 + bg = 0 \tag{1}$$

2. Let the r-coordinate of mass point 1 be $r = R_0 + \varepsilon$ and derive a differential equation about the infinitesimal displacement ε, then obtain the period of the small-amplitude oscillation.

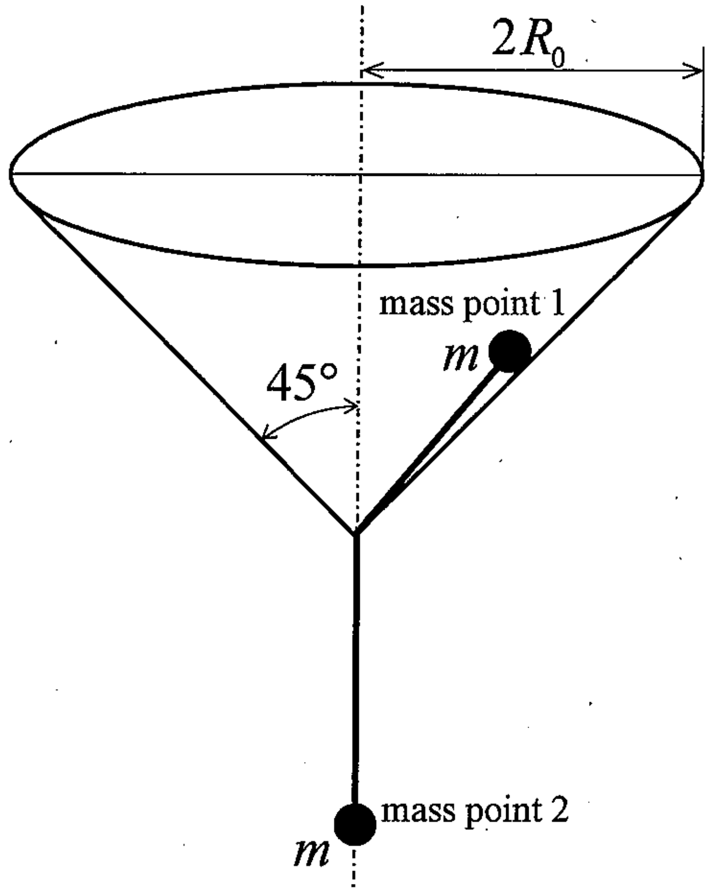

Figure 1.1

Problem 2

Suppose that a straight conductive rod (OP) with length a is rotating in the x-y plane with an angular velocity ω around its end point O in vacuum under a uniform magnetic flux density B_z in the $+z$ direction, as shown in Figure 2.1. The moment of inertia of the conductive rod around the rotating axis is denoted by J. Assuming that the thickness and electric resistance of the conductive rod and frictions can be ignored, answer the following questions.

I. Suppose that the conductive rod is rotating with a constant angular velocity $\omega = \omega_c$.

1. Obtain the area the conductive rod sweeps per unit time.
2. Using the answer above, obtain the voltage V between the end points O and P of the conductive rod.
3. List up all the electro-magnetic forces acting on electrons inside the conductive rod, and discuss the balance among them.

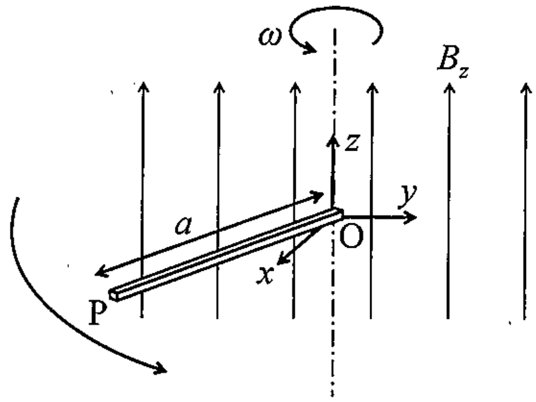

Figure 2.1

II. As shown in Figure 2.2, a circular conductive ring (radius a, centered at O) with no electric resistance is placed so that it always contacts with the conductive rod at P. Then, a resistor (electric resistance R) is connected between O and the ring. Here, ignore any magnetic fields generated by currents in the circuit other than the conductive rod.

1. The conductive rod rotates with a constant angular velocity ω_c, when a torque T_c is externally applied around the rotating axis. Obtain the torque T_c.

2. The rotation of the conductive rod distorts the uniform magnetic field. Draw schematically the magnetic flux lines around the conductive rod observed from point P toward O.

3. The angular velocity of the conductive rod at time $t = 0$ is ω_0, and no external torque is applied to it. Derive the equation that describes the temporal change of the angular velocity ω of the conductive rod, and plot ω against time t.

III. As shown in Figure 2.3, an inductor (inductance L) is connected in series with the circuit discussed in Question II.

1. Derive the differential equation that describes the angular velocity ω of the conductive rod.

2. The angular velocity ω shows an oscillatory behavior under a certain condition. In this situation, obtain the relation that the inductance L should satisfy.

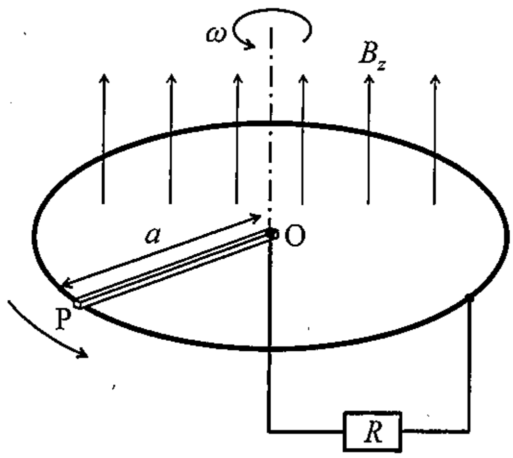

Figure 2.2

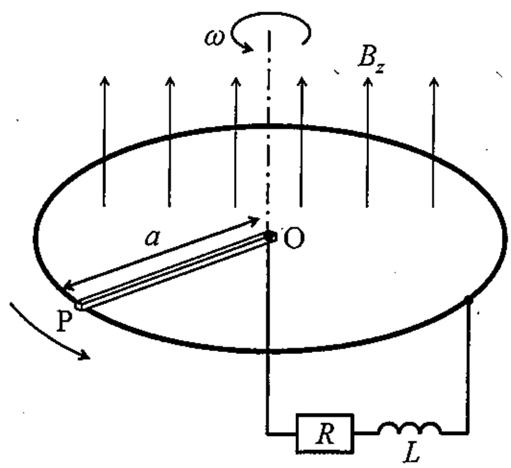

Figure 2.3

Problem 3

Consider various expansion processes of gases. Here, internal energy and volume per unit mole, absolute temperature, pressure and gas constant are denoted as U, V, T, p, and R, respectively. Molar specific heat at constant volume C_V and molar specific heat at constant pressure C_p are both constant regardless of the conditions.

I. Show that the following relation holds true for an ideal gas,

$$C_p - C_V = R. \tag{1}$$

II. Consider a quasi-static adiabatic expansion process or a quasi-static isothermal expansion process of an ideal gas of pressure p_0 and volume V_0. Draw schematically pressure-volume relations (i.e., adiabatic and isothermal lines) in a graph that clarifies the difference between the two processes.

Note that the following Poissons's equation is satisfied for a quasi-static adiabatic expansion process,

$$pV^{(C_p/C_V)} = \text{constant}. \tag{2}$$

III. Consider the process where an ideal gas undergoes adiabatic free expansion into vacuum. Show that the temperature of the gas after the expansion is the same as that before the expansion. Then, show that the process is irreversible.

IV. An ideal gas of volume V_1 at a temperature T_1 undergoes following two expansion processes A or B.

Process A: The gas undergoes an adiabatic free expansion to volume V_2. Then, after the gas is equilibrated, the gas undergoes a quasi-static adiabatic expansion to volume V_3.

Process B: The gas undergoes a quasi-static adiabatic expansion to volume V_2. Then, the gas undergoes an adiabatic free expansion to volume V_3.

The final temperatures of the two processes were found to be the same. Derive the relation among V_1, V_2, and V_3.

V. Consider an adiabatic free expansion process of a van der Waals gas that obeys the equation of state, $(p+a/V^2)(V-b)=RT$. Here, a and b are positive constants peculiar to the gas. The gas of volume V_4 undergoes adiabatic free expansion to volume V_5. Express the temperature difference ΔT, using all or part of V_4, V_5, R, a, b, and C_V. Then, explain the origin of the temperature change from the property of van der Waals gas.

The following relation may be used if necessary.

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p \qquad (3)$$

Problem 4

I. Consider the refraction of light (a plane wave) as shown in Figure 4.1. Light propagating in vacuum enters a homogeneous medium with a refractive index n at an angle of incidence θ_i and refracts at an angle of refraction θ_r. Here, the speeds of light in vacuum and in the medium are denoted by c and v, respectively. Answer the following questions.

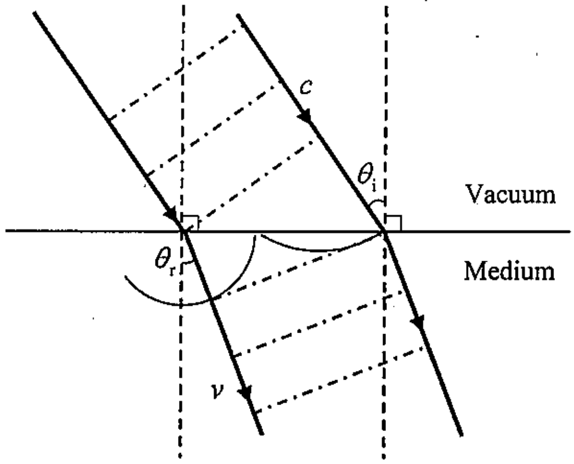

Figure 4.1

1. Derive the law of refraction (Snell's law) in terms of θ_i, θ_r, and n, using Huygens' principle.
2. In the quantum mechanical view, light energy is carried by photons. Using angular frequency ω and wavenumber k, the energy and momentum of a photon are denoted by $\hbar\omega$ and $\hbar k$, respectively (Here, $\hbar = h/2\pi$, h: Planck's constant). Express the relationship between the momentum $\hbar k_0$ in vacuum and the momentum $\hbar k_1$ in the medium, using all or part of θ_i, θ_r, c, v, and n.

II. Consider the refraction by a prism and the diffraction by an optical grating in vacuum. As shown in Figure 4.2, light (wavelength λ) enters the prism (refractive index n, apex angle α) at an angle of incidence θ_i and emerges at an angle of emergence θ_o. Next, as shown in Figure 4.3, light is normally incident to an optical grating composed of a large number of small prisms (refractive index n, apex angle β, width d) which are periodically arranged without any space. Answer the following questions.

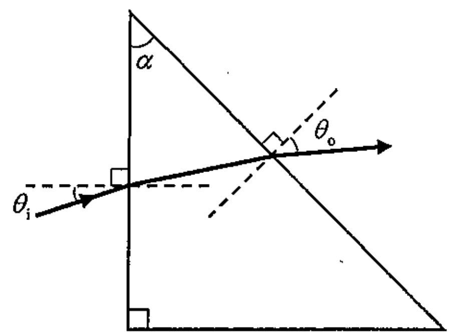

Figure 4.2

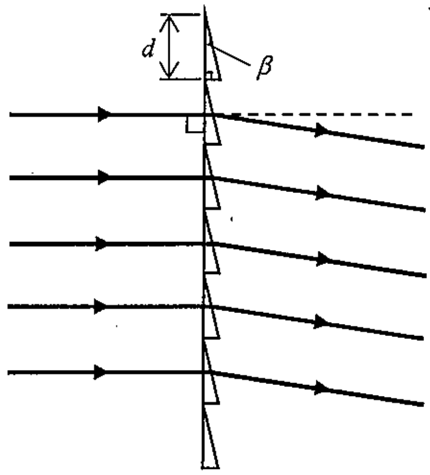

Figure 4.3

1. To determine the refractive index n of the prism, the angle of incidence θ_i and the angle of emergence θ_o are measured (see Figure 4.2). Express the refractive index n of the prism in terms of θ_i, θ_o, and α.
2. When the angle of incidence θ_i in Figure 4.2 is changed in the range of $0° \leq \theta_i < 90°$, light in a certain range of θ_i is not transmitted from the slant face but emerges from the bottom face. Express the range of θ_i in terms of α and n.
3. In Figure 4.3, when the first-order diffracted light from the optical grating and the refracted light by the small prisms propagate in the same direction, express the apex angle β of the small prisms in terms of d, n, and λ. Assume that the width d of the prisms is large enough compared to the wavelength λ of light and the apex angle β of the small prisms is sufficiently small.