2013
The Graduate School Entrance Examination
Physics
9:00 am－11:00 am

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Print your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of the answer sheet represent the problem number you answer (P 1, P 2, P 3, P 4) on that sheet and also the class of the master's course (M) and doctoral course (D) applicants. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks on each sheet with a pair of scissors.

6. You may use the blank sheets of the problem booklet as working space and for draft solutions, but you must not detach them.

7. Any answer sheet with marks or symbols irrelevant to your answers will be considered invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

Consider the motion of a sphere with a uniform density, radius a and mass M. Here, the sphere and the floors are assumed to be rigid and their deformations can be neglected. The x axis is horizontal and the y axis is vertical. The acceleration of gravity is denoted by g. An impulse shall work on the vertical plane (xy plane) including the center of mass G of the sphere. Answer the following questions.

I. Express the moment of inertia I_G for the sphere about the axis through the center of mass G, using a and M. Describe not only the answer but also the process by which you reached the answer.

II. As shown in Fig. 1.1, an impulse P acts in the x direction to the point positioned at height h on the sphere placed in a stationary position on floor 1, which is horizontal. Find height h_0 when the sphere rotates without sliding, assuming no friction between the sphere and floor 1.

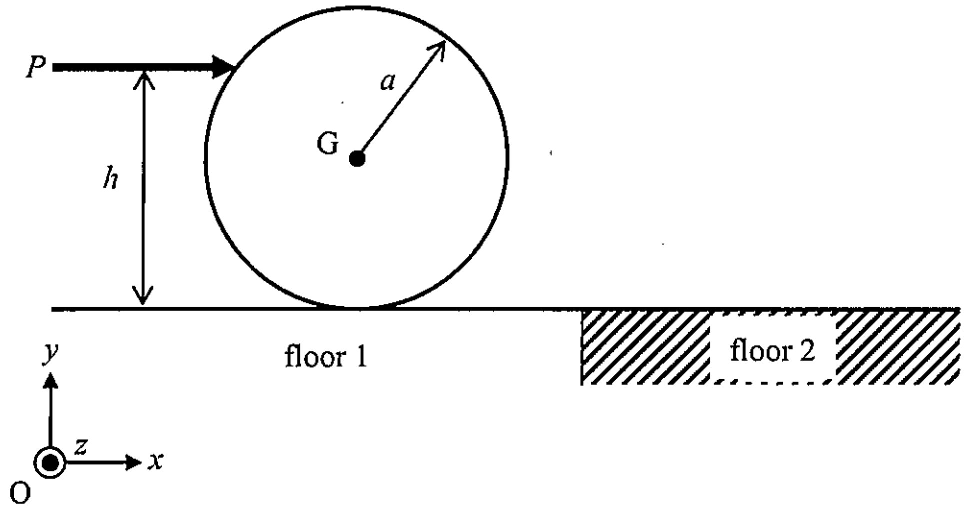
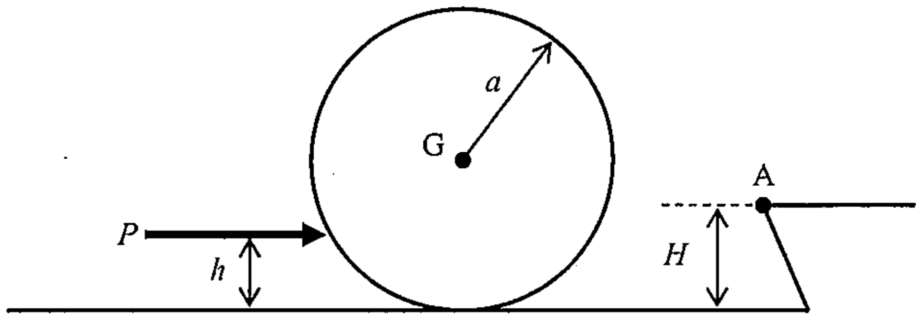


Figure 1.1

III. Next, an impulse P acts in the x direction to the point positioned at height $h = \frac{3}{2}a$ on the sphere placed in a stationary position on horizontal floor 1, as shown in Fig. 1.1. After the sphere starts moving, the sphere moves on to horizontal floor 2 at time $t = 0$. At $t = t_1$, the velocity of the center of mass G of the sphere becomes constant at v_1. Find time t_1 and velocity v_1, assuming that the coefficient of kinetic friction of floor 2 is constant and denoted by μ.

IV. As shown in Fig. 1.2, an impulse P acts in the x direction to the point positioned at height $h=\frac{1}{2}a$ on the sphere which is placed in a stationary position on the horizontal floor 1. The sphere cannot climb on to a step of height $H \geq H_m$. Find H_m, using the conservation law of angular momentum. Find the minimal impulse P_m that enables the sphere to climb on to the step, assuming that the sphere remains in contact with point A at the edge of the step and the contact is maintained during the climb without the sphere sliding backwards or forwards. Assume no friction between the sphere and floor 1.



floor 1

Figure 1.2

Problem 2

As shown in Fig. 2.1, a dielectric material (dielectric constant ε_1, magnetic permeability μ_1, electric conductivity $\sigma_1 = 0$) is attached to a conductive material (dielectric constant ε_2, magnetic permeability μ_2, electric conductivity σ_2), and an electromagnetic wave is propagated from the dielectric material to the conductive material.

Consider a plane electromagnetic wave (angular frequency ω) that has electric field $\boldsymbol{E}$ parallel to the x direction (E_x) and magnetic field $\boldsymbol{H}$ parallel to the y direction (H_y). The plane electromagnetic wave is propagated in the z direction, and is perpendicularly incident to the infinite and flat surface of the conductive material. As shown in Fig. 2.1, (E_x, H_y) of the incident wave (IN), the transmitted wave (T) and the reflected wave (R) are $(E_{\mathrm{IN}}, H_{\mathrm{IN}})$, $(E_{\mathrm{T}}, H_{\mathrm{T}})$ and $(E_{\mathrm{R}}, H_{\mathrm{R}})$, respectively. E_{IN} and H_{IN} at time t are $E_1 \exp(i\omega t - ikz)$ and $H_1 \exp(i\omega t - ikz)$, respectively.

Answer the following questions using the following Maxwell equations and telegraphic equations. Here, ε_1, ε_2, μ_1, μ_2 and σ_2 are real constants. Also, a, b and η are $(\varepsilon_1/\mu_1)^{1/2}$, $[\sigma_2/(\mu_2\omega)]^{1/2}$ and $(a/b)^{1/2}$, respectively. $z = 0$ corresponds to the surface of the conductive material, and the conductive material is placed in $z \geq 0$. k is a wave number, and i is an imaginary unit. Describe not only the answer but also the process by which you reached the answer.

$$\mathrm{rot}\boldsymbol{H} = \sigma\boldsymbol{E} + \varepsilon\frac{\partial \boldsymbol{E}}{\partial t}, \qquad (1)$$

$$\mathrm{rot}\boldsymbol{E} = -\mu\frac{\partial \boldsymbol{H}}{\partial t}, \qquad (2)$$

$$\frac{\partial^2 E_x}{\partial z^2} = \varepsilon\mu\frac{\partial^2 E_x}{\partial t^2} + \mu\sigma\frac{\partial E_x}{\partial t}, \qquad (3)$$

$$\frac{\partial^2 H_y}{\partial z^2} = \varepsilon\mu\frac{\partial^2 H_y}{\partial t^2} + \mu\sigma\frac{\partial H_y}{\partial t}. \qquad (4)$$

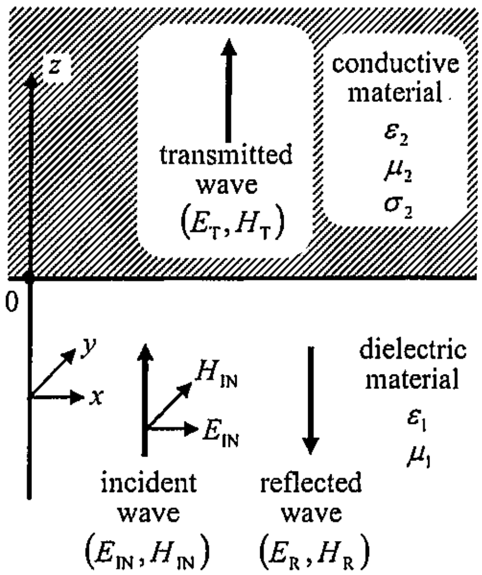


Figure 2.1

I. The magnitudes of E_R and H_R are E_3 and H_3, respectively. In the conductive material, $E_T = E_2 \exp[i\omega t - (\alpha + i\beta)z]$ and $H_T = H_2 \exp[i\omega t - (\alpha + i\beta)z]$ can be used under the condition of $\sigma_2 >> \varepsilon_2 \omega$. In addition, an approximation of $\alpha \approx \beta \approx (\mu_2 \sigma_2 \omega / 2)^{1/2}$ is used.

1. Express H_1 and H_3 using all or part of a, E_1 and E_3.
2. Express phase velocity v of the incident wave using ε_1 and μ_1.
3. Derive the relationship between E_2 and H_2. From the relationship, obtain the phase difference between E_T and H_T.

II. $\overline{\boldsymbol{S}}$ is a time average of a pointing vector $\boldsymbol{S}$ $(= \boldsymbol{E} \times \boldsymbol{H})$ and $|\overline{\boldsymbol{S}}|$ is a magnitude of $\overline{\boldsymbol{S}}$. By using the answers from Question I., answer the following questions.

1. $\boldsymbol{S}_{IN}$ and $\boldsymbol{S}_R$ are $\boldsymbol{S}$ of the incident wave and the reflected wave, respectively. Express $|\overline{\boldsymbol{S}_{IN}}|$ and $|\overline{\boldsymbol{S}_R}|$ using all or part of a, E_1 and E_3.
2. $\boldsymbol{S}_T$ is $\boldsymbol{S}$ of the transmitted wave. Express $|\overline{\boldsymbol{S}_T}|$ using all or part of b, α, E_2 and z. In addition, briefly explain the physical meaning of α implied from the expression of $|\overline{\boldsymbol{S}_T}|$.
3. Express both the reflectance and transmittance as a function of η, where reflectance is expressed by $|\overline{\boldsymbol{S}_R}| / |\overline{\boldsymbol{S}_{IN}}|$.

Problem 3

A sample consisting of 1 mol of gas molecules obeys the van der Waals equation expressed by Eq. (1). Consider the region in which pressure p, volume V and absolute temperature T are not in the range of liquid, and V is much larger than b.

$$\left(p+\frac{a}{V^2}\right)(V-b)=RT. \tag{1}$$

Here, R denotes the gas constant, and a and b are the positive constants related to the intermolecular force and the volume of molecules, respectively. When this gas with the internal energy U and the entropy S undergoes a reversible change, the following Eq. (2) can be obtained from the First Law of thermodynamics.

$$dU = TdS - pdV. \tag{2}$$

Answer the following questions. Here, the heat capacity at constant volume C_V of the gas is set to be constant.

I. Derive the following equation, which is one of the Maxwell relations.

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V. \tag{3}$$

The Helmholtz free energy $A = U - TS$ may be used if necessary.

II. When the gas is compressed or expanded at a constant temperature, the change in the internal energy of this gas is expressed by the following equation. Prove Eq. (4).

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p. \tag{4}$$

III. Consider a cycle composed of the following four reversible processes for a working fluid of 1 mol gas that obeys Eq. (1): an isothermal expansion at the constant absolute temperature T_2 (state A $\rightarrow$ state B), an adiabatic expansion (state B $\rightarrow$ state C), an isothermal compression at the constant absolute temperature T_1 (state C $\rightarrow$ state D), and an adiabatic compression (state D $\rightarrow$ state A). The volumes of the gas at states A, B, C and D are V_A, V_B, V_C and V_D, respectively.

This cycle is called a Carnot cycle. Figure 3.1 exhibits the Carnot cycle with the horizontal axis S and the vertical axis T, which shows these four processes. Express the heat transferred to the gas using V_A, V_B and T_2 in the process of the isothermal expansion from state A to state B. In addition, show the heat transferred to the gas using V_C, V_D and T_1 in the process of the isothermal compression from state C to state D.

IV. Show the temperature ratio T_1/T_2 in the cycle of Question III using V_B and V_C.

V. Derive the thermal efficiency η of the cycle of Question III, and compare it with the thermal efficiency when the working fluid is assumed to be an ideal gas.

VI. Consider a cycle composed of the following four reversible processes for a working fluid of 1 mol gas that obeys Eq. (1): an adiabatic compression (state E → state F), a heating at constant volume (state F → state G), an adiabatic expansion (state G → state H) and a cooling at constant volume (state H → state E). Here, T_E and T_F denote the absolute temperatures in states E and F, respectively. Also, the volumes of the gas at states E and F are V_E and V_F, respectively. Figure 3.2 exhibits this cycle with the horizontal axis S and the vertical axis T. The cycle can be approximated to be the sum of small Carnot cycles with an infinitely small area. Figure 3.3 indicates the expansion of one of the small cycles (state K → state L → state M → state J → state K) shown in Fig. 3.2. Prove that the thermal efficiencies of all the small cycles are equal. In addition, show the thermal efficiency of the entire cycle (state E → state F → state G → state H → state E) using ε and V_F, where $V_E/V_F = \varepsilon$.

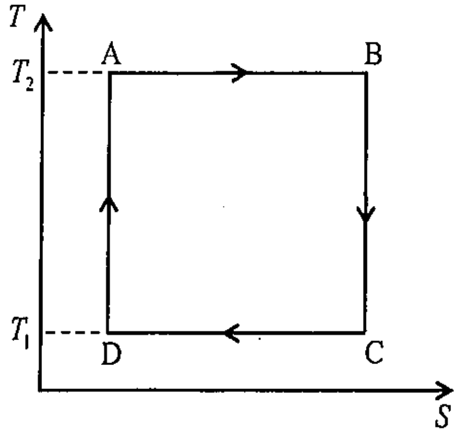


Figure 3.1

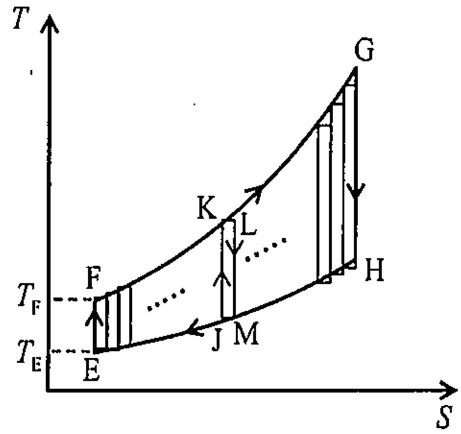


Figure 3.2

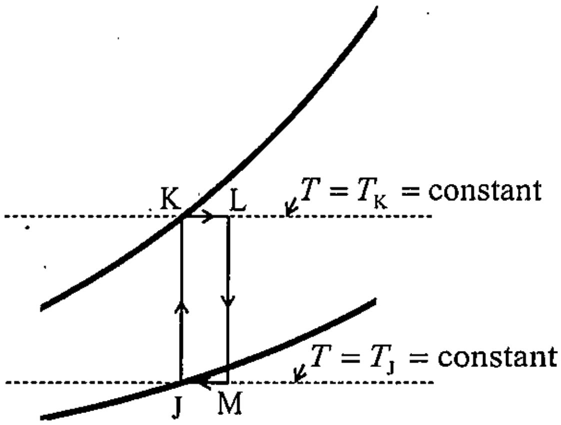


Figure 3.3

Problem 4

I. Light propagating in a homogeneous medium 1 with refractive index n_1 enters a homogeneous medium 2 with refractive index n_2. The angle of incidence is θ_1, and the angle of refraction is θ_2, as shown in Fig. 4.1. Answer the following questions.

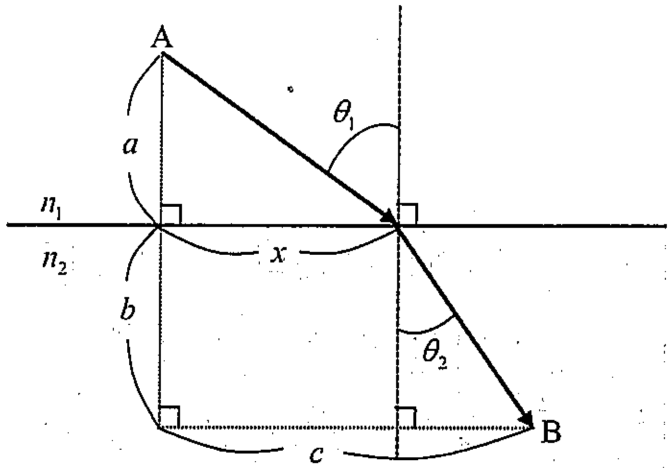


Figure 4.1

1. The product of the refractive index of the medium n, and the distance the light travels through this medium s, is called the optical path length, given by ns. Suppose the light travels from point A to point B as shown in Fig. 4.1. Express the optical path length from point A to point B, using n_1, n_2, a, b, c and x.
2. It is known that the actual path of the light makes the optical path length stationary (Fermat's principle). There is also a relationship between n_1, n_2, θ_1 and θ_2 as given by Eq. (1), known as Snell's law. Derive Eq. (1) using Fermat's principle.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 . \qquad (1)$$

II. Suppose propagating light in a vacuum enters a plane parallel glass plate at an incidence angle of θ_1 as shown in Fig. 4.2. The amount of shift between the incident light and the outgoing light is given by d. Answer the following questions.

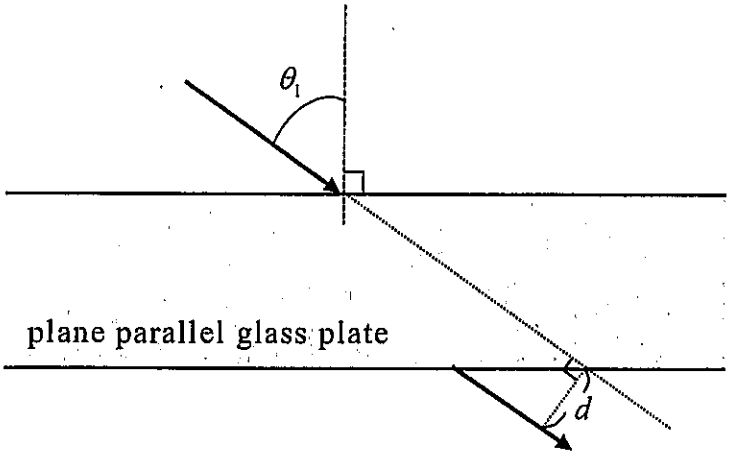


Figure 4.2

1. Consider a plane parallel glass plate 1 with refractive index n_1 and thickness h_1. Express the amount of shift d using all or part of θ_1, n_1 and h_1.
2. Another plane parallel glass plate 2 with refractive index n_2 and thickness h_2 is placed in direct contact with the underside of the glass plate 1 ($n_1 < n_2$), thus forming a laminated plane parallel glass plate. Express the amount of shift d caused by this laminated glass plate, using all or part of θ_1, n_1, n_2, h_1 and h_2.

III. Consider a method for determining the refractive index of a glass. Suppose propagating light in a vacuum enters a prism at an angle of incidence θ_a, and emerges from the prism at an outer angle of θ_b, as shown in Fig. 4.3. The refractive index of the glass used for the prism is n_1, and the angle between the two faces of the prism is α. Answer the following questions.

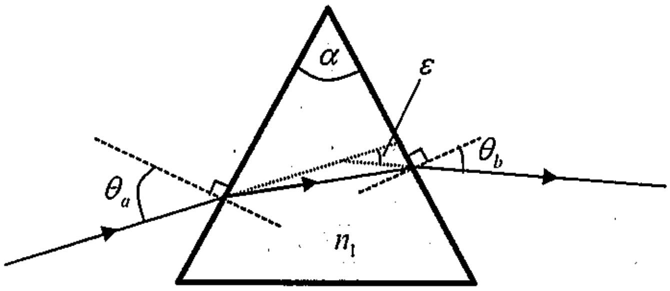


Figure 4.3

1. Express the angle of deviation ε using θ_a, θ_b and α.
2. Derive the relationship between θ_a and θ_b when ε has the minimal value $\varepsilon_{\min}$. Show the process by which you reached the answer.
3. Express the refractive index of the glass used for the prism n_1 using $\varepsilon_{\min}$ and α.