2020
The Graduate School Entrance Examination
Physics
1:00 pm－3:00 pm

GENERAL INSTRUCTIONS

Answers should be written in English or Japanese.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Fill in your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of each answer sheet represent the problem number that you answer (P 1, P 2, P 3, P 4) and also the class of the course (master M, doctor D) that you are applying. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks per sheet.

6. You may use the blank sheets of the problem booklets for rough papers without detaching them.

7. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.

8. You may not take the booklets or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

Consider an object A, consisting of a tube of outer radius $2r$ and inner radius r, and a solid cylinder of radius r that fits inside the tube. The tube and the solid cylinder share the same central axis as shown in Fig. 1.1. The tube and the solid cylinder are rigid bodies of uniform identical material. The tube has mass $3m$ and the solid cylinder has mass m. The amount of clearance between the inner surface of the tube and the outer surface of the solid cylinder can be ignored, and the solid cylinder can rotate inside the tube.

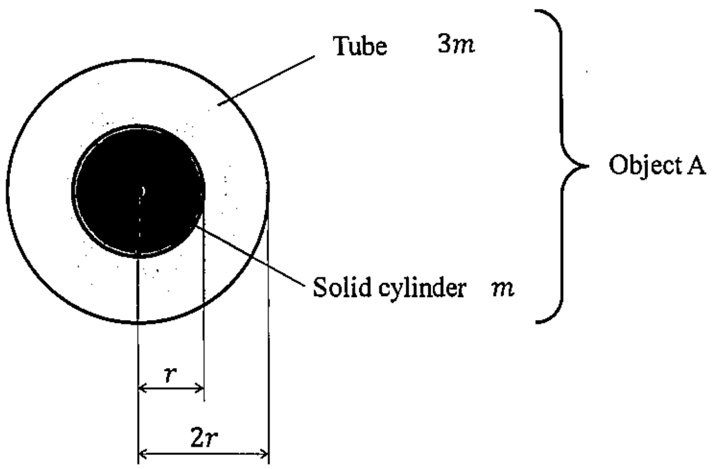

Figure 1.1

I. Derive the moment of inertia of the tube I_T and the solid cylinder I_C around the central axis, respectively. Also write the derivation process.

II. Consider a horizontal plane QR and an inclined plane (a slope) PQ at angle θ to the horizontal plane as shown in Fig. 1.2. The slope PQ has friction, and the horizontal plane QR has no friction. It is assumed that the object A can move between the horizontal plane and the slope while maintaining contact, and that there is no energy loss in transition from movement between the slope and the horizontal plane. The acceleration due to gravity is g.

In all of the following questions, the height of the central axis of the object A when on the horizontal plane is set as 0. You may use I_T and I_C as the moment of inertia of the tube and the solid cylinder around the central axis, respectively.

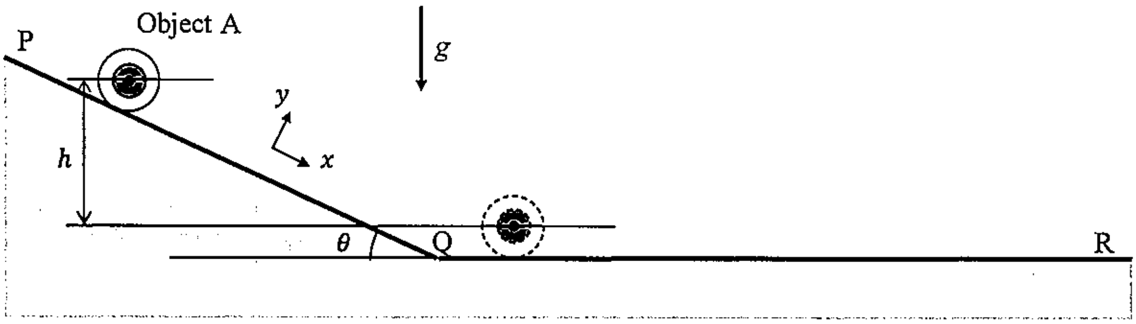

Figure 1.2

1. Consider the case where the friction between the inner surface of the tube and the outer surface of the solid cylinder can be ignored and the solid cylinder can rotate smoothly inside the tube around the same central axis. The object A was gently placed on the slope PQ so that the height of the central axis of the object A was h as shown in Fig. 1.2. Both the tube and the solid cylinder were not rotating. The tube then started rolling down the slope PQ smoothly without sliding.

   Obtain the translational velocity of the center of gravity of the object A in the horizontal direction v_1, the angular velocity of the tube around its central axis ω_{T1}, and the angular velocity of the solid cylinder around its central axis ω_{C1}, immediately after reaching the horizontal plane.

2. Next, consider the case where the solid cylinder can rotate inside the tube around the same central axis, but friction acts between the inner surface of the tube and the outer surface of the solid cylinder. The magnitude of the dynamic friction force between the tube and the solid cylinder is f. As shown in Fig. 1.2, when the object A was gently placed on the slope PQ so that the central axis of the object A was set at the height of h while neither the solid cylinder nor the tube was rotating, the object A started rolling smoothly down the slope PQ, without sliding. At this time, it was observed that the inner solid cylinder was slidingly rotating with a different angular velocity to the outer tube. Answer the following questions.

(i) Consider the motion of the object A on the slope PQ. As shown in the Fig. 1.2, consider a coordinate system in which the x axis is parallel to the slope PQ and the y axis is perpendicular. The translational velocity of the center of gravity of the object A along the slope is v, the angular velocity of the tube around its central axis is ω_T, the angular velocity of the solid cylinder around its central axis is ω_C, the x component and the y component of resultant force acting between the tube and the solid cylinder are N_x and N_y respectively, and normal force and friction force acting on the tube from the slope are N_{PQ} and F_{PQ} respectively.

① Show equations of motion of the tube's center of gravity motion in the x direction and the y direction and the tube's rotational equation of motion around the central axis of the tube. Also show the relation between v and ω_T.

② Show equations of motion of the solid cylinder's center of gravity motion in the x direction and the y direction and the solid cylinder's rotational equation of motion around the central axis of the solid cylinder.

(ii) Consider the motion of the object A on the horizontal plane QR. Answer the following questions by assuming that the translational velocity of the center of gravity of the object A along the horizontal plane, the angular velocity of the tube around its central axis, and the angular velocity of the solid cylinder around its central axis immediately after reaching the horizontal plane are v_Q, ω_{TQ}, and ω_{CQ}, respectively. While the object A traveled on the horizontal plane QR, the angular velocity of the tube around its central axis and that of the solid cylinder around its central axis eventually became equal.

① Derive the translational velocity of the center of gravity of the object A along the horizontal plane v_R and the angular velocity of the object A around its central axis ω_R.

② Derive the energy loss due to the dynamic friction between the inner surface of the tube and the outer surface of the solid cylinder during the movement of object A on the horizontal plane QR.

Problem 2

When an electric current flows through a conductor, the electric field and current density inside the conductor have spatial distributions depending on the frequency. Consider this phenomenon in the following questions. There is a conductor with conductivity σ filling the region $-h \le y \le h$ $(h \neq 0)$ in vacuum, as shown in Fig. 2.1. The conductor has infinite lengths in the x and z directions. An electric field E_z applied in the z direction generates electric current density $j_z = \sigma E_z$ in the conductor. The permittivity and magnetic permeability of the conductor are equal to the vacuum permittivity ε_0 and vacuum permeability μ_0, respectively. Due to symmetry, the magnetic flux density generated inside and outside the conductor by j_z has only an x component, $B_x(y)$. Positive E_z denotes an electric field in the $+z$ direction, and positive B_x denotes a magnetic flux density in the $+x$ direction.

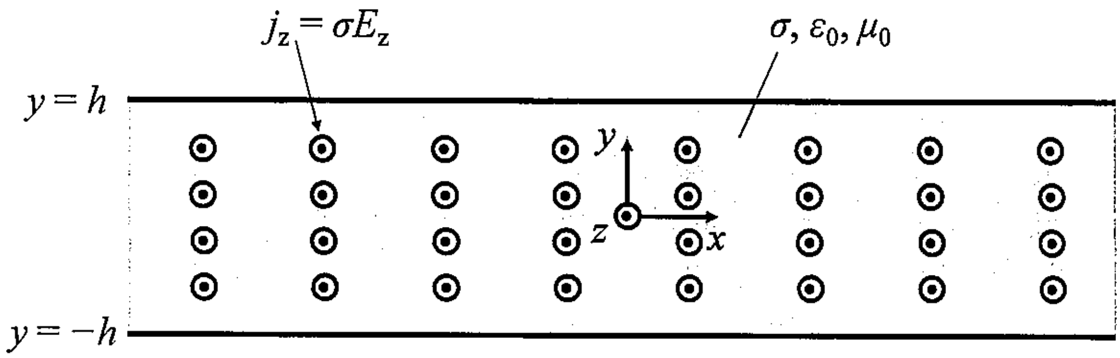

Figure 2.1

I. Consider the case where the electric field E_z is uniform and static.

1. Express the amount of heat generation per unit volume and unit time inside the conductor, using σ and j_z.
2. Express the magnetic flux density B_x as a function of y, for the region outside the conductor $y < -h$, inside the conductor $-h \le y \le h$, and outside the conductor $y > h$. Assume that $B_x(-y) = -B_x(y)$ because of its symmetry.

II. Consider the case where the electric field varies with time. In this case, the electric field and current density inside the conductor are not necessarily uniform. Both the electric field and the magnetic flux density oscillate with an angular frequency ω, and these are given by $E_z = \mathrm{Re}\{\tilde{E}\exp(i\omega t)\}$ and $B_x = \mathrm{Re}\{\tilde{B}\exp(i\omega t)\}$ with complex parameters $\tilde{E}$ and $\tilde{B}$. i is the imaginary unit.

1. Given the above electromagnetic properties of the conductor, Maxwell's equations inside the conductor contain the following two equations:

$$\frac{1}{\mu_0}\nabla \times \boldsymbol{B} - \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{j}, \tag{1}$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, \tag{2}$$

where $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{j}$ are the electric field, the magnetic flux density, and the current density, respectively. Prove that the complex electric field $\tilde{E}$ satisfies the following equation (3), based on equations (1) and (2). Assume that the x and y components of the electric field are zero, the y and z components of the magnetic flux density are zero, $\partial E_z/\partial x = \partial E_z/\partial z = 0$, and $\partial B_x/\partial x = \partial B_x/\partial z = 0$.

$$\frac{d^2\tilde{E}}{dy^2} + (\varepsilon_0\mu_0\omega^2 - i\omega\mu_0\sigma)\tilde{E} = 0. \tag{3}$$

2. When the conductor is a metal and the conductivity is sufficiently high ($\sigma \gg \varepsilon\omega$), equation (3) can be approximated as follows:

$$\frac{d^2\tilde{E}}{dy^2} - i\alpha^2\tilde{E} = 0, \tag{4}$$

where $\alpha = (\omega\mu_0\sigma)^{\frac{1}{2}}$. Find a general solution to this differential equation.

3. A current of $2h\mathrm{Re}\{j_c\exp(i\omega t)\}$ per unit length in the x direction flows in the conductor, where j_c is a real constant. The complex electric field satisfies the following equation.

$$\int_{-h}^{h} \sigma\tilde{E}\,dy = 2hj_c. \tag{5}$$

Assume that $\tilde{E}(-y) = \tilde{E}(y)$ because of its symmetry. Find the solution to equation (4) inside the conductor for these conditions. You can simplify the solution, using $\cosh(\beta) = (\exp(\beta) + \exp(-\beta))/2$ and $\sinh(\beta) = (\exp(\beta) - \exp(-\beta))/2$, where β is a complex number.

4. Inside the conductor, the squares of the electric field amplitudes at the middle ($y = 0$) and at the upper surface ($y \to h$) are denoted by $|\tilde{E}(0)|^2$ and $|\tilde{E}(h)|^2$, respectively.

(i) The angular frequency ω of the electric field is low ($\omega \to 0$). Choose the correct one from the following. Explain the reason.

a. $|\tilde{E}(0)|^2 \ll |\tilde{E}(h)|^2$

b. $|\tilde{E}(0)|^2 \approx |\tilde{E}(h)|^2$

c. $|\tilde{E}(0)|^2 \gg |\tilde{E}(h)|^2$

(ii) The angular frequency ω of the electric field is high ($\omega \gg \frac{1}{\mu_0 \sigma h^2}$). Choose the correct one from the following. Explain the reason.

a. $|\tilde{E}(0)|^2 \ll |\tilde{E}(h)|^2$

b. $|\tilde{E}(0)|^2 \approx |\tilde{E}(h)|^2$

c. $|\tilde{E}(0)|^2 \gg |\tilde{E}(h)|^2$

Problem 3

Consider a perfect gas defined by the equation of state (1) and a Van der Waals gas defined by the equation of state (2).

$$PV = nRT, \tag{1}$$
$$\left\{P + a\left(\frac{n}{V}\right)^2\right\}(V - nb) = nRT. \tag{2}$$

Here, P, V, n, R and T are the pressure, the volume, the amount of substance (number of moles), a gas constant and the thermodynamic temperature, respectively. Also, a and b are assumed to be constants. For a quasi-static process of these gases, the first law of thermodynamics is expressed as

$$dU = TdS - PdV, \tag{3}$$

where U and S are the internal energy and the entropy, respectively. Answer the following questions. Note that the specific heat at constant volume C_V is assumed to be constant and is expressed as

$$C_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V. \tag{4}$$

I. Consider the following relations for both a perfect gas and a Van der Waals gas. Using equation (3), $\left(\frac{\partial U}{\partial V}\right)_T$ is expressed as equation (5).

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P. \tag{5}$$

Using equation (5) and a Maxwell relation of $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$, a thermodynamic equation of state can be derived as expressed in equation (6) without using the entropy S, which we cannot measure directly.

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P. \tag{6}$$

Find $\left(\frac{\partial U}{\partial V}\right)_T$ for both a perfect gas and a Van der Waals gas.

II. Consider real gas effects during quasi-static expansions. A gas of a unit amount of substance (1 mol), pressure P_0, volume V_0, and thermodynamic temperature T_0 is inside a cylinder fitted with a piston as the initial condition. Assume that the system is thermally isolated from the external environment. Answer the following questions.

1. The gas undergoes an adiabatic reversible expansion to a volume of $2V_0$ from V_0 by displacement of the piston as shown in Fig. 3.1. Find the thermodynamic temperatures T and the entropy changes ΔS for both a perfect gas and a Van der Waals gas after such an expansion.

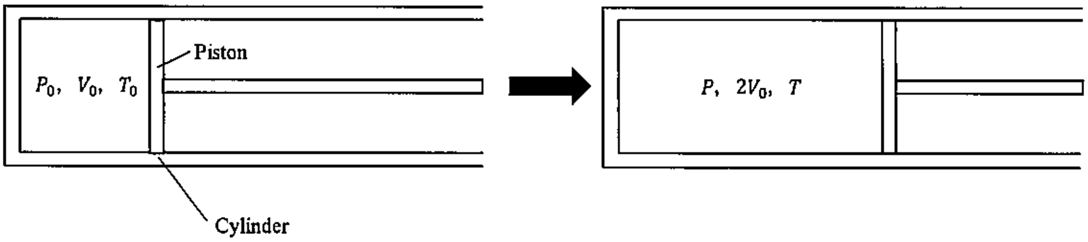

Figure 3.1

2. Next, consider that the gas in the cylinder is heated by a heater and is reversibly expanded to a volume of $2\ V_0$ from V_0 at the constant thermodynamic temperature of T_0 as shown in Fig. 3.2. Find the changes in the internal energy and the entropy, ΔU and ΔS, for both a perfect gas and a Van der Waals gas after such an expansion. Also, explain the reason for the difference of the change in the internal energy ΔU between a perfect gas and a Van der Waals gas.

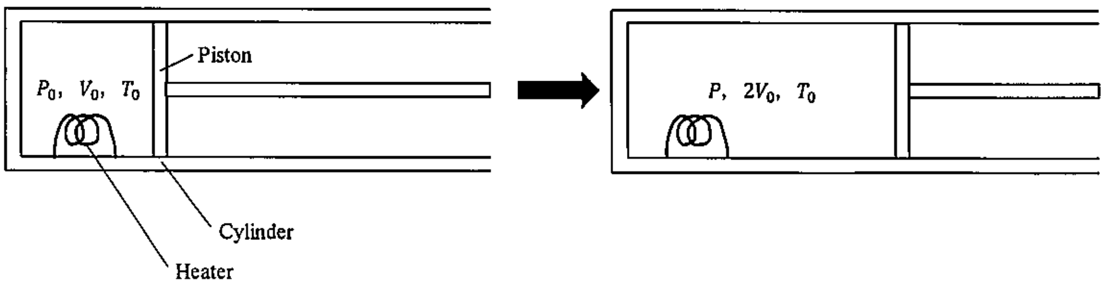

Figure 3.2

III. Chamber A of volume V_0 and Chamber B of volume V_0 are connected by a valve as shown in Fig. 3.3. At the initial condition, a perfect gas with pressure P_0, volume V_0, and thermodynamic temperature T_0 is in Chamber A. The gas consists of a unit amount of substance (1 mol). Chamber B is evacuated. Consider expansion of the gas in Chamber A by opening the valve. Answer the following questions, assuming that the system is thermally isolated from the external environment.

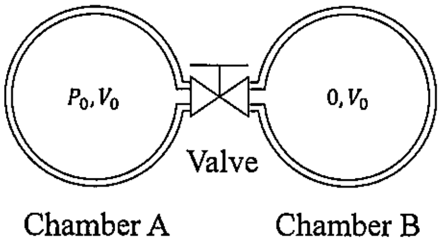

Figure 3.3

1. Find the thermodynamic function (quantity of state) which remains constant during this process.
2. Find the thermodynamic temperature T and the entropy change ΔS after such an expansion.
3. Explain about the irreversibility of this process, with reasoning.

Problem 4

A rod of length L is fixed at both ends by support A and support B as shown in Fig. 4.1. Assume that moments are not applied at both ends of the rods.

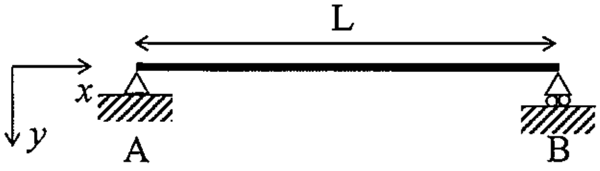

Figure 4.1

I. Consider the bending deformation of the rod when a distributed load $q(x)$ per unit length is applied to the rod in the y direction. Assume that the deformation of the rod is infinitesimally small, and consider only displacement in the y direction (the downward direction is positive as shown in Fig. 4.2). The mass of the rod is negligible. Answer the following questions.

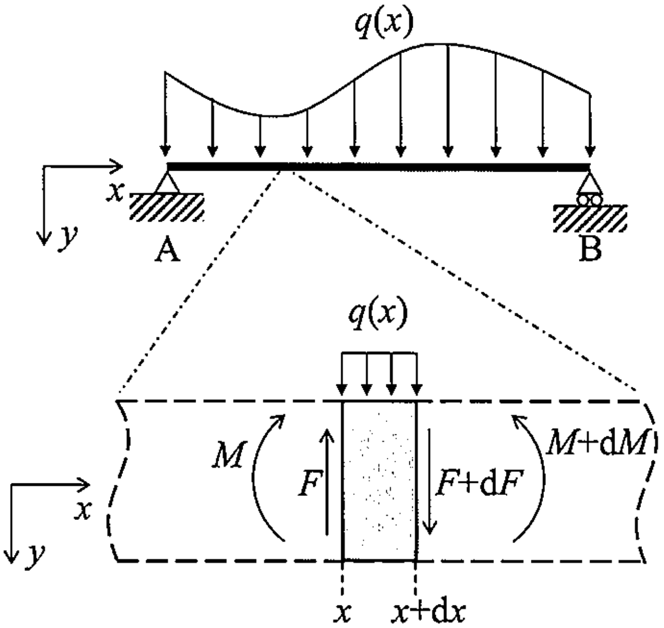

Figure 4.2

1. Consider an element of the rod at any position x. Assume that the force F and the moment M act on the cross section of the rod to maintain static equilibrium as shown in Fig. 4.2.

   (i) Show that equation (1) holds true between the force F and the distributed load $q(x)$ in this case.

   $$\frac{dF}{dx} = -q(x). \tag{1}$$

   (ii) Show that equation (2) holds true between the moment M and the force F in this case.

   $$\frac{dM}{dx} = F. \tag{2}$$

2. When the deformation of the rod is infinitesimally small, it is assumed that the rod is deformed only by the moment M. In this case, equation (3) holds true between the displacement y, and the moment M. Here, R is assumed to be a constant. When the distributed load $q(x) = \mathrm{k}$ is applied (k is a constant), derive the maximum displacement of the rod. Here, assume that $y = 0$ at both ends of the rod.

   $$\mathrm{R}\frac{d^2y}{dx^2} = -M. \tag{3}$$

II. Next, consider the bending vibration of the rod shown in Fig. 4.1. Here, consider about free vibration. Assume that the rod is uniform, the density of the rod is ρ, and the cross sectional area is S. Also, assume that the deformation of the rod is infinitesimally small, and consider only motion in the y direction. Gravitational forces are negligible. Answer the following questions.

1. Consider the motion of an element of the rod as shown in Fig. 4.3. By considering the equation of motion of the element in the figure, and by using equations (2) and (3), show that the equation of motion of the bending vibration of the rod in the y direction is expressed as equation (4). Assume that the element moves only in the y direction, and that the force in the x direction and the rotation by the moment are negligible.

$$\mathrm{R}\frac{\partial^4 y}{\partial x^4} + \rho \mathrm{S}\frac{\partial^2 y}{\partial t^2} = 0. \tag{4}$$

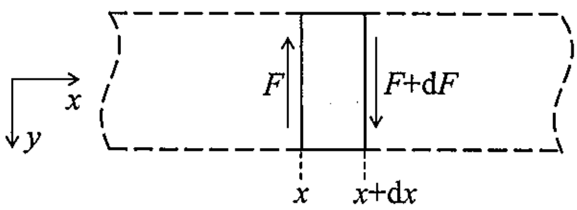

Figure 4.3

2. Assume that the solution of equation (4) obtained in Question II. 1 is $y(x,t) = X(x)\exp(i\omega t)$, where i is the imaginary unit. Show that the general solution $X(x)$ of equation (4) is expressed by equation (5). Here, $\mu^4 = \frac{\rho S \omega^2}{\mathrm{R}}$, and $\mathrm{C}_1 \sim \mathrm{C}_4$ are constants. Also, $\exp(ix) = \cos x + i\sin x$, $\sinh x = \left(\exp(x) - \exp(-x)\right)/2$, and $\cosh x = \left(\exp(x) + \exp(-x)\right)/2$.

$$X(x) = \mathrm{C}_1 \sin \mu x + \mathrm{C}_2 \cos \mu x + \mathrm{C}_3 \sinh \mu x + \mathrm{C}_4 \cosh \mu x. \tag{5}$$

3. In the case when the rod is fixed at both ends similar to Fig. 4.1, $y = 0$ at both ends of the rod.
   (i) Obtain the possible values of μ using equation (5).
   (ii) Explain the behavior of the bending vibration based on the result of Question II. 3. (i).

問 題 訂 正

科目名：物理学

第２問　II. 1.　５行目（５ページ）

（誤）… それぞれ電場，磁場，電流密度である。

（正）… それぞれ電場，磁束密度，電流密度である。

No correction in the English version.

第２問　II. 2.　１行目（５ページ）

（誤）… 導電率が十分に高い場合($\sigma \gg \underline{\varepsilon}\omega$),

（正）… 導電率が十分に高い場合($\sigma \gg \underline{\varepsilon_0}\omega$),

Problem 2　II. 2.　Line 1-2　(Page 6)

(incorrect)... the conductivity is sufficiently high ($\sigma \gg \underline{\varepsilon}\omega$),

(correct)... the conductivity is sufficiently high ($\sigma \gg \underline{\varepsilon_0}\omega$),

2019 年 8 月 27 日

工学系研究科

大学院入試委員会本部