2012
The Graduate School Entrance Examination
Physics
9:00 am－11:00 am

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets whether in English or Japanese until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Print your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of the answer sheet represent the problem number you answer (P 1, P 2, P 3, P 4) on that sheet and also the class of the master's course (M) and doctoral course (D) applicants. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks per sheet with a pair of scissors.

6. You may use the blank sheets of the problem booklet as working space and for draft solutions, but you must not detach them.

7. Any answer sheet with marks or symbols irrelevant to your answers will be considered invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

I. As shown in Fig. 1.1, rods OA, AB, and BC whose mass is uniformly distributed are linked by pins A and B without friction, and the two ends of the entire frame are fixed on the ground using frictionless pin supports O and C, respectively. Pin A is suspended from the ceiling using rope AD whose weight is negligible. Rods OA and AB are horizontal and rod BC is vertical. Rods OA and BC have length r and mass m, while rod AB has length $2r$ and mass $2m$. The weight of pins A and B is negligible, and gravitational acceleration is g. In this situation, suppose rope AD is cut. Answer the following questions about the rod motions at this moment. Here, the xyz coordinate system fixed on the ground is defined as shown in Fig. 1.1, and unit vectors in the directions of x, y, and z axes are $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$, respectively. If necessary, you may use the fact that the moment of inertia of a rod with length of r and mass of m rotating around its center of gravity is $\frac{1}{12}mr^2$.

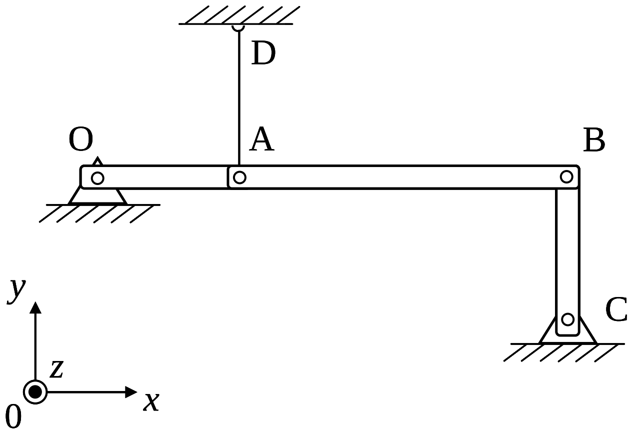

Figure 1.1

1. Given the angular acceleration of rod OA in the counter-clock-wise direction is α_A, express the acceleration of pin A with respect to the ground as a vector.
2. Given the angular acceleration of rod AB in the counter-clock-wise direction is α_B, express the acceleration of pin B

with respect to pin A, and the acceleration of pin B with respect to the ground, as vectors.

3. Express the angular acceleration α_B with α_A.
4. Considering the dynamic equilibrium of rod OA, determine the vector of the force exerted by rod OA on rod AB at point A, in terms of α_A, r, m, and g.
5. Considering the dynamic equilibrium of rod AB, express the vector of the force exerted by rod AB on rod BC at point B, and α_A, using r, m, and g, as necessary.

II. For the same frame that is shown in Fig. 1.1, consider the case that rod OA is rotating at a constant angular velocity ω_o in the clock-wise direction as shown in Fig. 1.2. Answer the following questions.

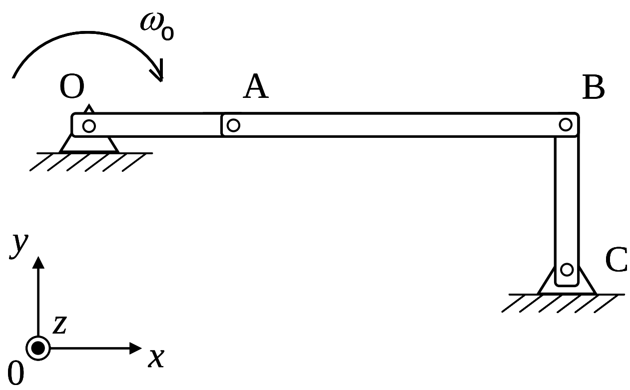

Figure 1.2

1. Express the acceleration of pin B with respect to the ground and the angular acceleration of rod BC, as vectors, using ω_o and r, as necessary.
2. Considering the rotational dynamic equilibrium of rod BC, determine the x component of the force exerted by rod AB on rod BC at point B, in terms of ω_o, r, and m.
3. Considering the dynamic equilibrium of rod AB, express the vector of the force exerted by rod OA on rod AB at point A, and the y component of the force exerted by rod AB on rod BC at point B, using ω_o, r, m, and g, as necessary.

Problem 2

Consider concentric cylindrical hollow conductors A and B, which are long enough. As shown in Fig. 2.1, the outer radius of A is a and the inner radius of B is b. The region between the conductors A and B is filled with gas, whose permittivity is ε. The conductor A is connected to a voltage source and the conductor B is grounded. Let r be the distance from the center of the cylinders.

If necessary, use the following expression for the vector operators in cylindrical coordinates:

$$\nabla \cdot \boldsymbol{A} = \frac{1}{r}\frac{\partial (rA_r)}{\partial r} + \frac{1}{r}\frac{\partial (A_\varphi)}{\partial \varphi} + \frac{\partial (A_z)}{\partial z}, \tag{1}$$

$$\nabla^2 f = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial f}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{\partial^2 f}{\partial z^2}. \tag{2}$$

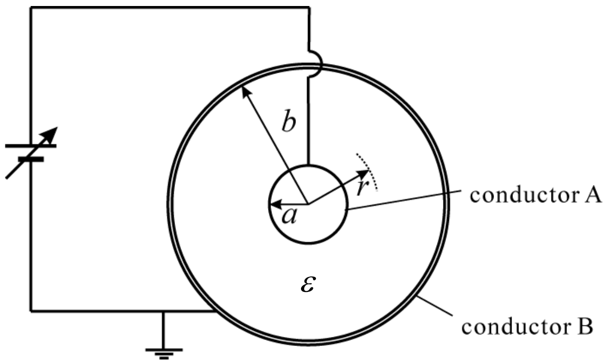

Figure 2.1

I. Voltage $V_0 (V_0 \geq 0)$ is applied to the conductor A.

1. Derive the potential and the electric field strength at r ($a < r < b$).
2. Derive the strength and the direction of the force per unit surface area exerted on the conductor A.

II. When the voltage applied to the conductor A is increased and exceeds a threshold value, ions are generated continuously due to ionization of the gas at the surface of the conductor A as shown in Fig. 2.2. The ions move toward the conductor B at the velocity $\boldsymbol{v} = \mu \boldsymbol{E}$, where μ is a constant, and a current flows between the conductors A and B. Let the electric field strength at the surface of the conductor A in this situation be E_i, and the current per unit axial length between the conductors be I.

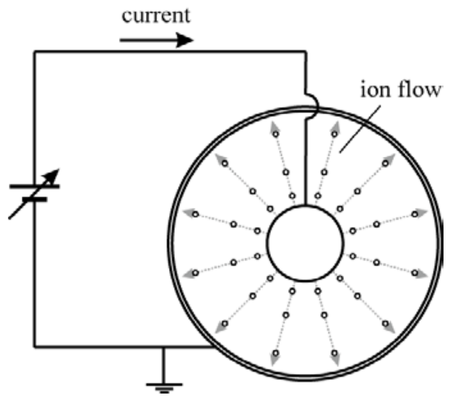

Figure 2.2

1. Express the ion charge density $\rho(r)$ at $r\,(a < r < b)$ using the electric field strength $E(r)$.
2. Derive a differential equation for $E(r)$.
3. Solve the differential equation for $E(r)$. You may wish to use the substitution $G(r) = \{E(r)\}^2$.
4. In general, E_i can be considered to be constant regardless of the voltage applied on the conductor A. Explain briefly how the electric field distribution between the conductors A and B changes from the electrostatic one by the occurrence of the ion flow.

Problem 3

An ideal gas is expressed by the equation of state $PV = nRT$, where P is the pressure, V is the volume, n is the number of moles, R is the gas constant, and T is the temperature. With the Boltzmann constant k and Avogadro's number N_{A}, the following relation exists.

$$R = kN_{\mathrm{A}}. \tag{1}$$

The formula $Z \equiv \dfrac{PV}{nRT}$ is the definition of the compressibility factor Z, which is equal to 1 for an ideal gas. The compressibility factor Z for a non-ideal gas is expanded in a power series in $\dfrac{n}{V}$, known as the Virial expansion,

$$Z = 1 + \frac{n}{V}B(T) + \left(\frac{n}{V}\right)^2 C(T) + \left(\frac{n}{V}\right)^3 D(T) + \bullet\bullet\bullet, \tag{2}$$

where $B(T)$, $C(T)$, and $D(T)$ are the second, the third, and the fourth Virial coefficients, respectively.

I. The second Virial coefficient $B(T)$ depends on the temperature T and is related to the intermolecular potential $\phi(r)$ as follows,

$$B(T) = 2\pi N_{\mathrm{A}} \int_0^{\infty} \left[1 - \exp\left(-\frac{\phi(r)}{kT}\right)\right] r^2 \,\mathrm{d}r. \tag{3}$$

Derive the second Virial coefficient $B(T)$ when the dependence of the intermolecular potential $\phi(r)$ on molecular distance r is given as follows,

$$r < \sigma \quad : \phi(r) = \infty, \tag{4}$$

$$r \geq \sigma \quad : \phi(r) = -\varepsilon_0 \left(\frac{\sigma}{r}\right)^6, \tag{5}$$

where σ and ε_0 are positive constants. The condition, under which the gas phase is maintained, can be used if necessary.

II. The van der Waals equation of state expresses a non-ideal gas as

$$\left(P+\frac{an^2}{V^2}\right)\left(\frac{V}{n}-b\right)=RT, \tag{6}$$

where parameters a and b are positive constants specific to the gas. Derive the second Virial coefficient $B(T)$ and the third Virial coefficient $C(T)$ by expanding the above equation.

III. Obtain the parameters a and b by comparing the second Virial coefficients derived in I. and II.

IV. Explain the physical meanings of the parameters a and b, and the constants σ and ε_0.

Problem 4

I. Consider a particle with an energy E and a mass m in a one-dimensional potential step shown in Fig. 4.1.

$$V(x) = \begin{cases} 0 & (x < 0\text{: region 1}) \\ -V_0 & (x \geq 0\text{: region 2}) \end{cases}. \tag{1}$$

Here, $V_0 > 0$. The particle enters the potential step at $x = 0$ from the region 1. The wavefunctions for the particle satisfy the following Schrödinger equations:

$$\begin{cases} \text{region 1} & -\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_1(x)}{dx^2} = E\,\psi_1(x), \\ \text{region 2} & -\dfrac{\hbar^2}{2m}\dfrac{d^2\psi_2(x)}{dx^2} - V_0\,\psi_2(x) = E\,\psi_2(x), \end{cases} \tag{2}$$

where $\psi_1(x)$ and $\psi_2(x)$ are the wavefunctions in the regions 1 and 2, respectively, and $\hbar$ is given by $\hbar = h/2\pi$ (h: Planck constant). Answer the following questions.

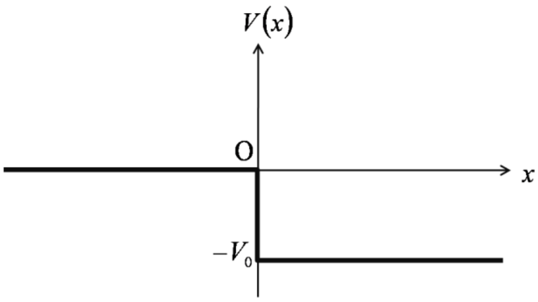

Figure 4.1

1. Suppose that the wavefunction for the incident particle is expressed as e^{ik_1x} $(k_1 > 0)$. The particle is transmitted through the potential step, while the reflection partially takes place. When the wavenumber in the region 2 is expressed as k_2, derive the wavefunctions ψ_1 and ψ_2 using k_1 and k_2. In addition, express k_1 and k_2 in terms of E.
2. Derive the reflectance R and the transmittance T for the particle.

II. Consider the potential well shown in Fig. 4.2.

$$V(x) = \begin{cases} 0 & (x < 0\text{: region 1}) \\ -V_0 & (0 \le x \le a\text{: region 2}). \\ 0 & (x > a\text{: region 3}) \end{cases} \tag{3}$$

Here, $V_0 > 0$ and $a > 0$. As in the case of I., a particle, having an energy E and a mass m, enters the potential well from the region 1. The wavefunction for the incident particle is expressed as $e^{ik_1 x}$ $(k_1 > 0)$. Answer the following questions.

1. Obtain the transmittance for the particle from the region 1 to the region 3.
2. Derive the energies for the particle, which give the maximum transmittance.
3. For the particle showing the maximum transmittance, calculate the wavenumber k_1 in the region 1. Briefly explain the physical meaning of the result of the calculation in relation to the well width a.

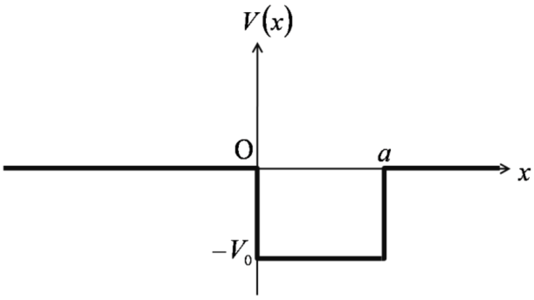

Figure 4.2