2017
The Graduate School Entrance Examination
Physics
9:00 am－11:00 am

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Fill in your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of the answer sheet represent the problem number you answer (P 1, P 2, P 3, P 4) on that sheet and also the class of the master's course (M) and doctoral course (D) applicants. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks on each sheet with scissors.

6. You may use the blank sheets of the problem booklet for rough papers without detaching them.

7. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

As depicted in Figure 1.1, a slender uniform rigid rod AB of negligible thickness, length l and mass M, rests vertically on a horizontal plane. The initial position of A is defined as the origin. Axes x and y are defined as shown. When a negligibly small velocity is given to the upper end B in the positive x direction, the rod starts to tilt. Let the center of gravity of the rod be G, and denote the acceleration of gravity as g. Answer the following questions. Neglect friction from the horizontal plane and air. For questions I to IV, you can assume that the rod end A maintains contact with the horizontal plane.

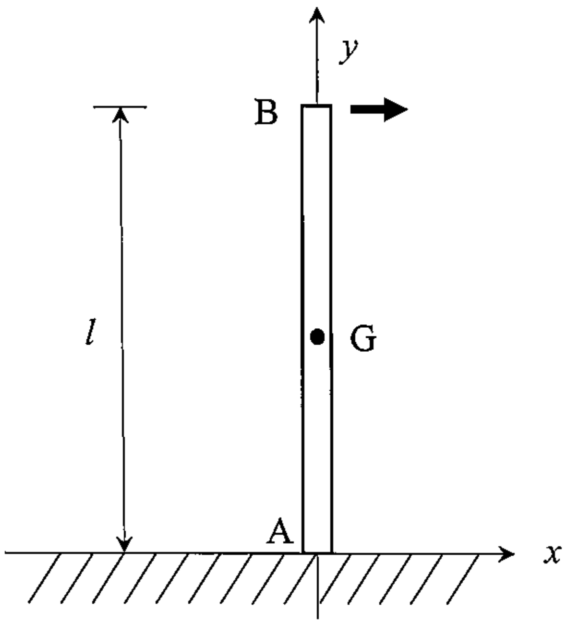

Figure 1.1

I. Show that the moment of inertia I of the rod about the axis normal to the xy-plane going through G can be expressed as $I = \frac{1}{12}Ml^2$.

II. Let θ be the angle between rod AB and the y axis. Angle θ increases with time from $\theta = 0$. Derive the equations of motion of the rod for translational and rotational motions about G. Here, normal force R acts on the rod from the horizontal plane at A.

III. Derive the differential equation for the angle θ. The equation should only include g and l.

IV. Obtain the angular velocity of the rod about G and the velocity of B just before B touches the horizontal plane.

V. Prove that the rod end A maintains contact with the horizontal plane until B touches the horizontal plane.

Problem 2

As shown in Figure 2.1, two semicircular conductive plates having radius r are held parallel to each other with distance z_0 in a vacuum environment. Permittivity of vacuum is ε_0. Let the upper and lower conductive plates be Electrode A and Electrode B, respectively. The center of the straight edges of Electrode A and Electrode B are named O_A and O_B. Electrode A can rotate around Point O_A. Let the angle formed by the two straight edges of the electrodes be θ, where $\theta = 0$ when there is no overlap between the electrodes. In this problem, consider only the electric field perpendicular to the electrode planes, and ignore the edge effects. Answer the following questions.

I. Let the angle of electrodes θ be $\pi/2$. True charges Q and $-Q$ $(Q > 0)$ are put on Electrode A and Electrode B, respectively.

1. Obtain the electric field intensity E in the area where the electrodes are overlapping.
2. Obtain the electric potential V_1 of Electrode A to Electrode B.
3. Obtain the capacitance C between Electrode A and Electrode B.
4. Angle θ is increased slowly from $\theta = \pi/10$ to $\theta = 19\pi/10$. Derive the potential V of Electrode A to Electrode B as a function of angle θ, and draw a graph of V versus θ.

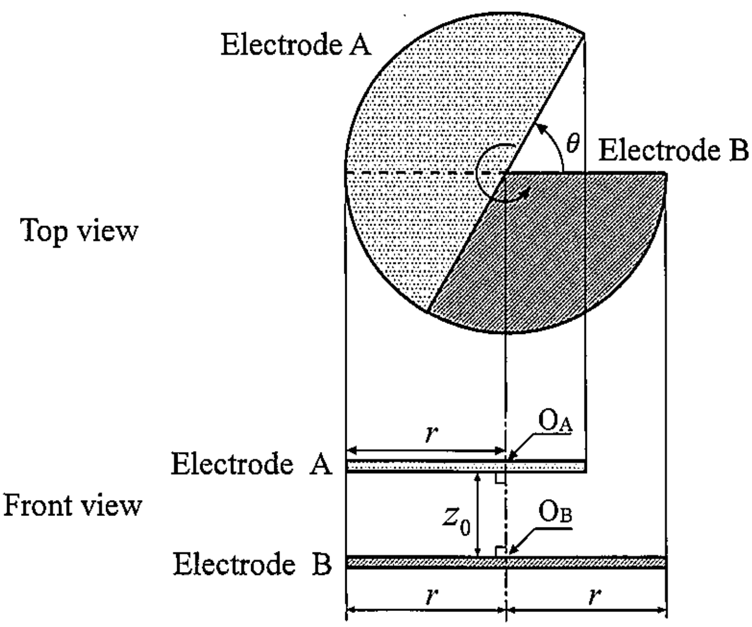

Figure 2.1

II. Charges on Electrode A and Electrode B are set to 0. Angle θ is set to $\theta = \pi$. As shown in Figure 2.2, a semicircular Dielectric C having radius r, thickness $z_0/2$ and relative permittivity k is placed directly over Electrode B. While grounding only Electrode B, true charge is fixed on the upper surface of Dielectric C with a uniform surface density $\sigma\ (\sigma > 0)$. Polarization occurs in Dielectric C and the electric field within the dielectric becomes smaller.

1. Draw electric force lines and electric flux lines in two separate front views. When drawing, assume $k = 2$ and let density of the lines reflect the values of $\varepsilon_0 E$ and electric flux density D.
2. Obtain true charges Q_A and Q_B on Electrode A and Electrode B.

III. Following the previous question, Electrode A is also connected to the ground as shown in Figure 2.3.

1. Let the induced true charges on Electrodes A and B be Q'_A and Q'_B, and the intensity of electric field between the upper surface of Dielectric C and Electrode A be E'_A and that in Dielectric C be E'_B. Express E'_A and E'_B, respectively, as functions of Q'_A and Q'_B.
2. Derive the relationship between E'_A and E'_B, and obtain Q'_A and Q'_B.
3. Draw electric force lines and electric flux lines in two separate front views. When drawing, assume $k = 2$ and let density of the lines reflect the values of $\varepsilon_0 E$ and electric flux density D.
4. Next, θ is set to $\theta = 0$. Electrode A is slowly rotated with a constant angular velocity $\omega\ (\omega > 0)$. Derive the current flowing into Electrode A as a function of time t, $I(t)$, while $0 < \theta < 2\pi$.

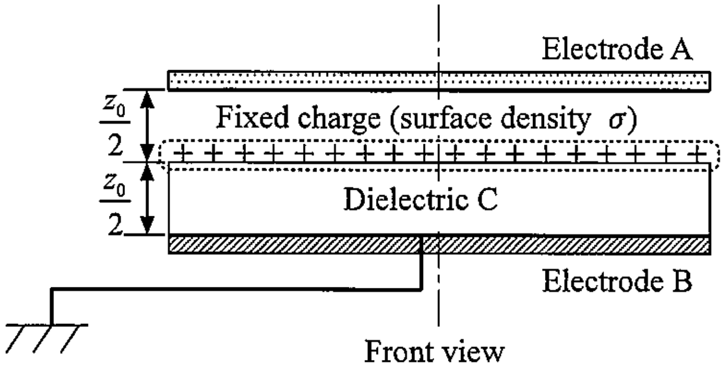

Figure 2.2

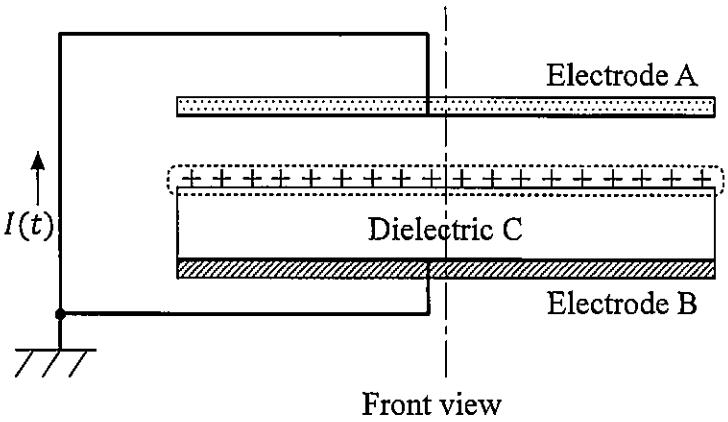

Figure 2.3

Problem 3

I. The following equation of state is applicable for an ideal gas,

$$pv = RT\,. \tag{1}$$

Here, p is the pressure, v is the volume per unit mass or specific volume, T is the temperature, and R is a constant determined by the gas species. Answer the following questions.

1. The coefficient of volumetric expansion α and the isothermal compressibility k_T are given by,

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p, \tag{2}$$

and

$$k_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T. \tag{3}$$

For the case of an ideal gas, express α and k_T using any one of the state quantities of Equation (1). Here, subscripts p and T mean that the pressure p and the temperature T are kept constant.

2. Show that the difference between the specific heat at constant pressure c_p and the specific heat at constant volume c_v is expressed as,

$$c_p - c_v = \frac{vT\alpha^2}{k_T}, \tag{4}$$

for gases in general, including an ideal gas. Here, c_p and c_v are given by,

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p, \tag{5}$$

and

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v, \tag{6}$$

where s is the entropy per unit mass. Subscript v denotes that specific volume v is kept constant. Use the following Maxwell's equations,

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T, \tag{7}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T, \tag{8}$$

and the chain rule,

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1. \tag{9}$$

3. Consider a system composed of an ideal gas, where the state of the system changes quasi-statically in a reversible process from the thermodynamic equilibrium State 1 (p_1, v_1, T_1) to another thermodynamic equilibrium State 2 (p_2, v_2, T_2). Express the change in s using c_v, v_1, v_2, R, T_1, and T_2. Assume c_v to be constant.
4. Assuming that the reversible change from State 1 to State 2 is adiabatic in the previous question I.3, express T_2/T_1, using v_1, v_2, and the ratio of specific heats $\kappa = c_p/c_v$.

II. Consider the thermal efficiency of a quasi-static cycle for an ideal gas. Here, q_A, q_B, q_C, q_D are heat supplied per unit mass, and q_E, q_F, q_G are heat exhausted per unit mass. Assume that the specific heat at constant pressure c_p and the specific heat at constant volume c_v are constant. Answer the following questions.

1. Cycle A depicted as a $p-v$ diagram in Figure 3.1, consists of four reversible processes; adiabatic change 1→2, isovolumetric change

$2 \to 2'$, adiabatic change $2' \to 4$, and isovolumetric change $4 \to 1$. Express the thermal efficiency $(q_A - q_E)/q_A$ of this cycle, using the compression ratio $\varepsilon = v_1 / v_2$ and the ratio of specific heats $\kappa = c_p / c_v$.

2. Cycle B depicted in Figure 3.2, consists of four reversible processes; adiabatic change $1 \to 2$, isobaric change $2 \to 3$, adiabatic change $3 \to 4$, and isovolumetric change $4 \to 1$. Express the thermal efficiency $(q_B - q_F)/q_B$ of this cycle, using ε, κ, and the cut-off ratio $\sigma = v_3 / v_2$.
3. Cycle C depicted in Figure 3.3, consists of five reversible processes; adiabatic change $1 \to 2$, isovolumetric change $2 \to 2'$, isobaric change $2' \to 3$, adiabatic change $3 \to 4$, and isovolumetric change $4 \to 1$. Express the thermal efficiency $(q_C + q_D - q_G)/(q_C + q_D)$ of this cycle, using ε, κ, σ, and the pressure rise ratio $\rho = p_3 / p_2$.
4. Among the three cycles A, B and C mentioned above, name the cycles with the largest and the smallest thermal efficiency under the same compression ratio ε. Give reasons. Let $\varepsilon > 2$, $\kappa = 4/3$, $\rho > 1$, $\sigma = 2$, and $2^{1/3} = 1.26$.

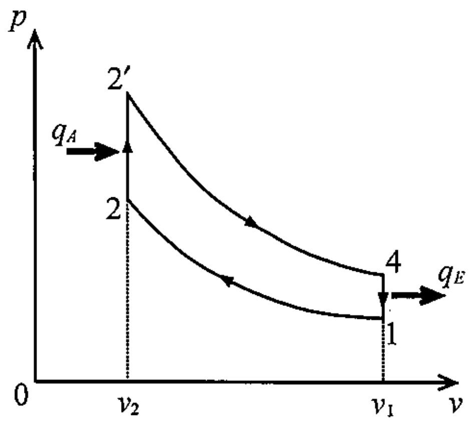

Figure 3.1

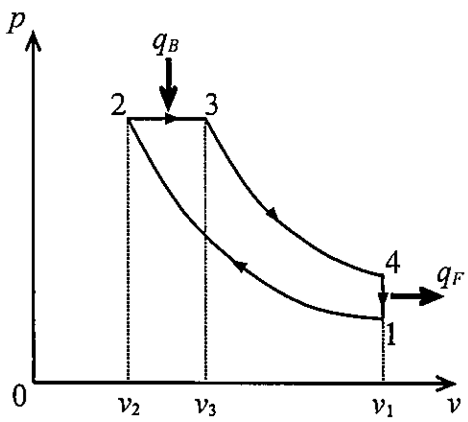

Figure 3.2

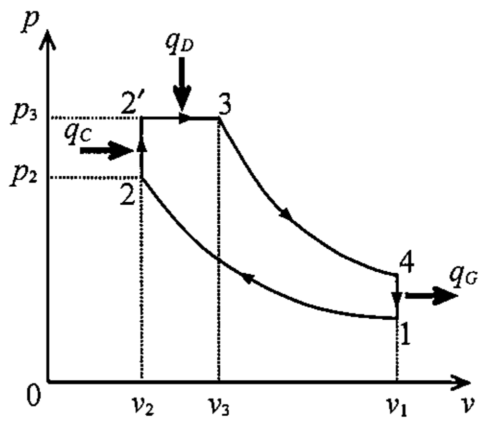

Figure 3.3

Problem 4

I. Consider the vibration in a one dimensional ring lattice consisting of N atoms with mass M_0, interconnected by springs with spring constant K_s. N is sufficiently large, and neighboring atoms can locally be modeled to be in a straight line as shown in Figure 4.1. The distance between the neighboring atoms at equilibrium is given as a. Answer the following questions.

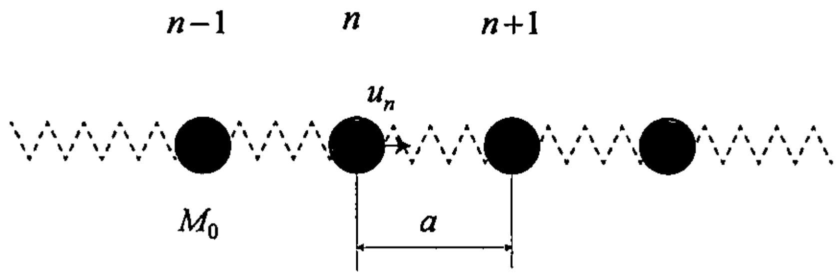

Figure 4.1

1. Displacement of the n^{th} atom from its equilibrium is denoted as u_n. Express the equation of motion of the atom using M_0, K_s, u_n, u_{n+1}, and u_{n-1}. Force and displacement are defined to be positive in the rightward direction in Figure 4.1.
2. The general solution of the equation of motion in question I.1 is given as,

$$u_n = u\exp\{-i(\omega t - kna)\}, \qquad (1)$$

where u is the amplitude of vibration of each atom, ω is the angular frequency, and k is the wavenumber. Using this general solution, derive the equation giving the relation between ω and ka.

Next, consider the vibration in a one dimensional ring lattice consisting of two species of atoms with masses M_1 and M_2, alternately interconnected with springs of spring constant K_s as shown in Figure 4.2. The atoms can locally be modeled to be in a straight line. The distance between the neighboring atoms at equilibrium is given as a. Answer the following questions.

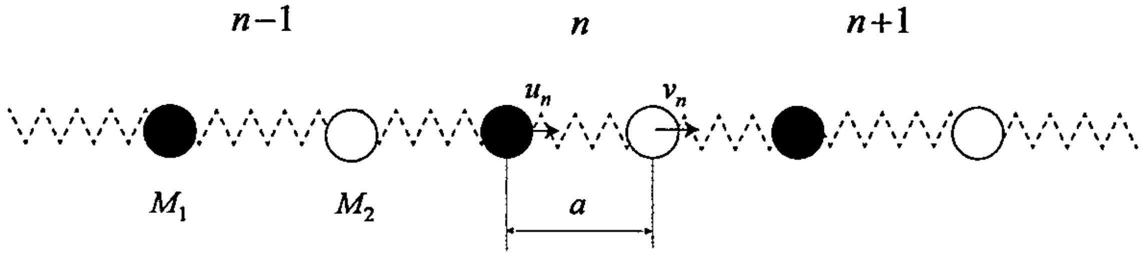

Figure 4.2

3. Displacement of the n^{th} atom with mass M_1 from its equilibrium is denoted as u_n, and that with mass M_2 as v_n. Express the two equations of motion for the two atoms using M_1, M_2, K_s, u_n, u_{n+1}, u_{n-1}, v_n, v_{n+1}, and v_{n-1}. Force and displacement are defined to be positive in the rightward direction in Figure 4.2.
4. Derive the general solution for the equation obtained in the previous question. Here, let the amplitudes of u_n and v_n be u and v, respectively.
5. Derive the equation giving the relation between ω^2 and ka.

II. In quantum mechanics, the harmonic oscillation for a one dimensional lattice as introduced in question I is given as the solution of the following Schrödinger equation,

$$\left\{-\frac{\hbar^2}{2M_0}\frac{d^2}{dx^2}+\frac{1}{2}M_0\omega^2x^2\right\}\varphi(x)=E\varphi(x), \qquad (2)$$

where $\varphi(x)$ is the wavefunction, x is the atomic position, E is the eigenvalue, and $\hbar = h/2\pi$ where h is the Planck's constant. The wavefunctions of the ground state and the first excited state of this Schrödinger equation are given by,

$$\varphi_0(x)=C_0\exp\left(-\frac{M_0\omega}{2\hbar}x^2\right), \qquad (3)$$

$$\varphi_1(x)=C_1\sqrt{\frac{M_0\omega}{\hbar}}\,x\exp\left(-\frac{M_0\omega}{2\hbar}x^2\right), \qquad (4)$$

where C_0 and C_1 are the normalizing constants.

1. Derive the eigenvalue for the ground state E_0 and that for the first excited state E_1 using $\varphi_0(x)$ and $\varphi_1(x)$.
2. Show that the expectation values of the atomic position $\langle x \rangle$ and momentum $\langle p \rangle$ become 0 both in the ground state and the first excited state.