2016
The Graduate School Entrance Examination
Physics
9:00 am − 11:00 am

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Print your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of the answer sheet represent the problem number you answer (P 1, P 2, P 3, P 4) on that sheet and also the class of the master's course (M) and doctoral course (D) applicants. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks on each sheet with a pair of scissors.

6. You may use the blank sheets of the problem booklet as working space and for draft solutions, but you must not detach them.

7. Any answer sheet with marks or symbols irrelevant to your answers will be considered invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

Consider a sphere S1 of radius r and mass m having a uniform volume density, a sphere S2 of radius r and mass $3m$ having a uniform volume density, and a spherical shell S3 of radius r and mass m having infinitesimally thin thickness with the uniform area density. As shown in Figure 1.1, when one of the above spheres (the sphere or the spherical shell) is first placed at Point A on a rough slope which is inclined to the horizontal plane by an angle θ, and then gently released, the sphere starts rolling down along the slope. Denoting the acceleration due to gravity as g, the gravitational force acts at the center of the sphere O, and in addition the normal force N and the friction force F act at the contact point P between the sphere and the slope. Angular velocity around O is denoted as ω, where the counter-clock-wise direction is defined as positive. Answer the following questions.

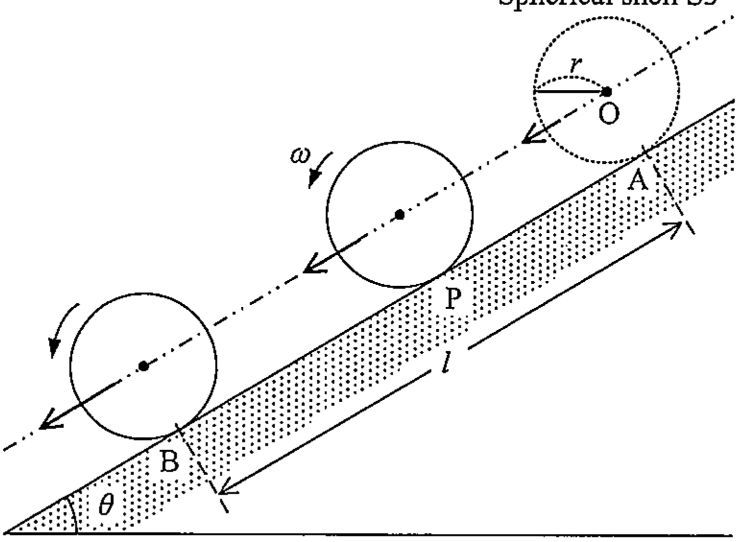

Figure 1.1

I. Derive the moment of inertia I_1 and I_3 about the axis through the center of sphere, respectively for Sphere S1, and for Spherical shell S3. Describe not only the answer but also the process by which you arrive at the answer.

II. Consider the case where Sphere S1 rolls down without slipping. The static friction coefficient between the sphere and the slope is hereafter denoted as μ_0.

1. Write down the following equations: the equation of motion for the center of gravity of the sphere, the equation of force balance in the direction perpendicular to the slope, and the equation of rotational motion around the center of gravity.
2. Obtain the acceleration of the center of gravity when Sphere S1 is rolling down along the slope. In addition, find the relation between μ_0 and θ when Sphere S1 rotates without slipping.

III. Consider the case where slippage occurs while Sphere S1 rolls down the slope. When Sphere S1 is gently released from Point A on the slope, it starts rolling down with slipping. In this case, denoting the coefficient of kinetic friction between Sphere S1 and the slope as μ ($<\mu_0$), the relation $F = \mu N$ holds. Taking the moment of release as time $t = 0$, write down the relative slipping velocity $q(t)$ of Sphere S1 with respect to the slope at the contact point P, as a function of time.

IV. Under conditions where no slippage occurs, Spheres S1, S2 and S3 are released gently from Point A separately, so as to begin rolling down along the slope. Consider Point B located at a distance l down the slope from Point A. Denoting the time required for each of the spheres S1, S2, and S3 to travel from Point A to Point B as T_1, T_2, and T_3 respectively, calculate T_2 / T_1 and T_3 / T_1.

Problem 2

As shown in Figure 2.1, a rod of radius a and height h ($h >> a$), made of a material having electrical conductivity σ and dielectric constant ε, is placed in vacuum, and is equipped with a pair of electrodes on the top and the bottom. Straight conductors are extended from the top and the bottom electrodes, in line with the center axis of the rod, and wired to a current source and a switch at a far distance; hence the electric or the magnetic field produced by the current source, the switch, and the wiring to them can be neglected. The resistance of the electrodes and the wirings are also negligible. We ignore the electric field concentration on the electrode edges, so that the electric field can be assumed to be uniform, confined inside the rod, in the direction parallel to the rod axis. The dielectric constant of vacuum is denoted as ε_0, and the rod's magnetic permeability is equal to that of vacuum μ_0.

Answer the questions about the electric and magnetic fields created inside and outside of the rod when an electrical current flows from the top electrode to the bottom electrode.

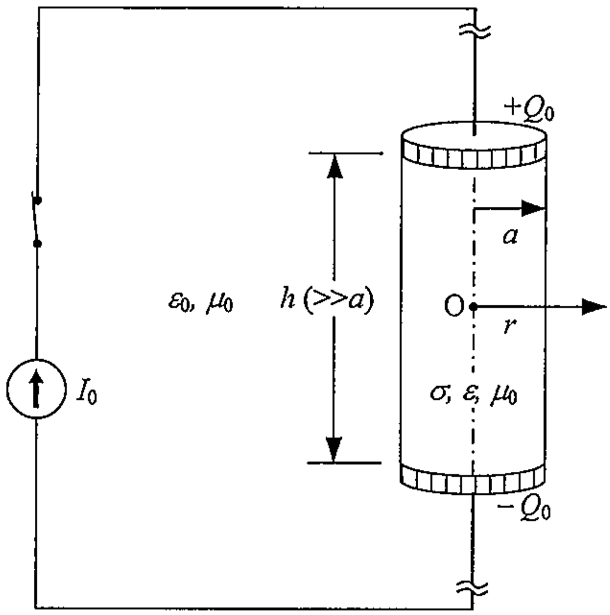

Figure 2.1

I. When a constant current I_0 flows through the rod, the top and the bottom electrodes develop electrical charges $+Q_0$ and $-Q_0$, respectively. You can assume that the electrical charge distribution is uniform on the electrodes.

1. Find the electrical resistance R between the top and the bottom electrodes.
2. Express the relationship between the electrical current density j and the electric field E in the rod.
3. Derive the relationship between the current I_0 and the electrical charge Q_0 using the Gauss's law.
4. Find the electrostatic capacitance C between the electrodes when the constant current I_0 flows.
5. Express the magnetic flux density as a function of the distance r measured from the center of the rod O and perpendicular to the axis of the rod. Consider both inside ($r \leq a$) and outside ($r > a$) the rod separately.

II. When the switch is opened at time $t = 0$, the conduction current inside the rod decays as a function of time, which can be expressed as follows.

$$I(t) = I_0 \exp\left(-\frac{t}{RC}\right) \quad (1)$$

Assuming that the current distribution in the rod is uniform, answer the following questions.

1. Express the displacement current in the rod, as a function of time.
2. Which way (upward or downward) is the direction of the displacement current? Give reasons.
3. Express the magnetic flux density as a function of the distance r measured from the center of the rod O and perpendicular to the axis of the rod. Consider both inside ($r \leq a$) and outside ($r > a$) the rod separately.

Problem 3

Consider heat processes where the working fluid is an ideal gas (gas constant is R). The molar specific heat at constant volume C_V, as well as the molar specific heat at constant pressure C_P, are assumed to be constant regardless of conditions. Assuming all state changes are quasi-static, answer the following questions. In all cases, describe the process with which you arrive at your answers.

I. We take the working fluid of one mole. We shall denote the pressure as P, the absolute temperature T, the volume V, the internal energy U, and the entropy S.

1. Prove the following formula (1).

$$dU = C_V\, dT \tag{1}$$

2. Find the entropy change dS caused by the temperature change dT and the pressure change dP.
3. Let S_0 be the entropy at the absolute temperature T_0 and the pressure P_0. Find the entropy S at the absolute temperature T and the pressure P.

II. As shown in Figure 3.1, gasses with different pressure, each in its equilibrium state, are separated by a diaphragm and contained in a thermally insulated container. Let this be State 'a'. In this state, the volume, the pressure, and the mole number of the gases (gas 1 and gas 2) in each compartment separated by the diaphragm are denoted as V_1, V_2, P_1, P_2, n_1 and n_2, respectively. It is also assumed that both compartments are maintained at the same temperature T_a. When the diaphragm is ruptured, the two gasses mix without chemical reactions and finally reach another equilibrium, which we shall call State 'b'.

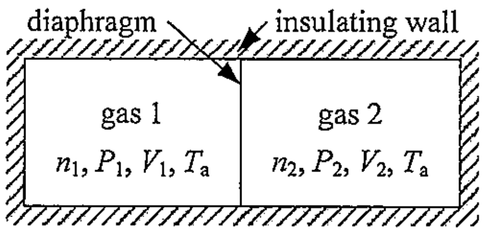

Figure 3.1

1. Express the pressure of the gas P_b in terms of P_1, P_2, n_1 and n_2, after the equilibrium of State 'b' is reached.
2. Find the entropy difference of the gas between State 'a' and State 'b', and show that the process from State 'a' to State 'b' is irreversible.

III. Answer the following questions about the Carnot cycle whose P-V (pressure-volume) diagram is depicted in Figure 3.2.

1. Does the entropy increase or decrease, during the process from State B to State C ? Answer the question with reasons.
2. Draw the T-S (temperature-entropy) diagram.
3. Derive the efficiency of the Carnot cycle using the T-S diagram.

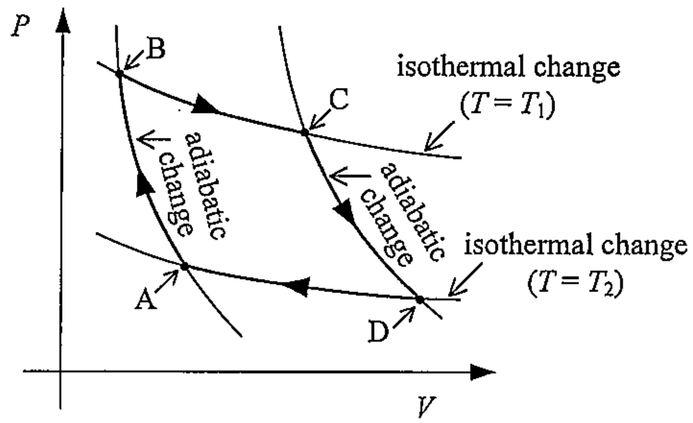

Figure 3.2

Problem 4

Consider a particle of mass m that vibrates along the x axis under a periodic external force $mf\cos\omega t$ (f is a positive constant, ω denotes angular frequency and t time). The equation of motion of this particle can be written as $m\ddot{x}+2m\mu\dot{x}+m\omega_0^2 x=mf\cos\omega t$ ($\ddot{x}=d^2x/dt^2$). Here, $-m\omega_0^2 x$ is the restoring force (the force proportional to the distance x from the origin O, ω_0 is the characteristic angular frequency) and $-2m\mu\dot{x}$ is the resisting force (the force proportional to $\dot{x}=dx/dt$, μ is a positive constant). The position of the particle $x_S(t)$ in the steady state, i.e., after a sufficient time has elapsed, can be written using its initial phase α as follows:

$$x_S(t)=\frac{f}{\sqrt{(\omega_0^2-\omega^2)^2+4\mu^2\omega^2}}\cos(\omega t-\alpha),\quad \tan\alpha=\frac{2\mu\omega}{\omega_0^2-\omega^2}. \qquad (1)$$

$x_S(t)$ can also be written using the elastic amplitude A_e and the absorption amplitude A_a as

$$x_S(t)=A_e\cos\omega t+A_a\sin\omega t. \qquad (2)$$

In solving the problems below, the following formulae about the integrals over one period (from time τ to $\tau+T$) may be used, where T ($T=2\pi/\omega$) is the period of oscillation.

$$\frac{1}{T}\int_\tau^{\tau+T}\cos^2\omega t\,dt=\frac{1}{2},\quad \frac{1}{T}\int_\tau^{\tau+T}\sin^2\omega t\,dt=\frac{1}{2},\quad \frac{1}{T}\int_\tau^{\tau+T}\sin\omega t\cdot\cos\omega t\,dt=0. \qquad (3)$$

I. Answer the following questions concerning the steady state oscillation. Here, $p(t)$ is the power supplied by the external force, and P is the time average of $p(t)$ over one period. In addition, $w(t)$ is the total mechanical energy of the particle, and W is the time average of $w(t)$ over one period.

1. Write A_e and A_a using ω, ω_0, μ, and f.
2. Making use of the definition that $p(t)$ is given by a product of the external force and $\dot{x}$, write P using all or part of m, f, ω, and A_a. In addition, briefly explain its physical interpretation in terms of the energy supplied and the energy consumed.

3. Write W using all or part of m, f, ω, ω_0, and μ.
4. P takes the maximum value P_{max} at $\omega = \omega_0$. Derive the full width at half maximum $\Delta\omega$ ($\Delta\omega = \omega_+ - \omega_-$) from the two points of ω (ω_+ and ω_-, $\omega_+ > \omega_-$) at which P becomes $P_{max}/2$ in the ω dependence of P.

II. Consider how a transient, caused by the external force starting to act at $t = 0$, finally reaches the steady state. The position of this particle $x_T(t)$ is given by

$$x_T(t) = \exp(-\mu t)(B_1 \cos\omega_1 t + B_2 \sin\omega_1 t) + x_S(t), \tag{4}$$

where B_1 and B_2 are constants, and $\omega_1 = (\omega_0^2 - \mu^2)^{1/2}$. Answer the following questions when the resisting force is sufficiently small ($\omega_0 > \omega_1 >> \mu$). Here, the initial conditions are $x_T(0) = 0$ and $\dot{x}_T(0) = 0$.

1. Write B_1 and B_2 using all or part of ω, ω_1, μ, A_e, and A_a.
2. Derive an approximate solution for $x_T(t)$ in the resonant condition ($\omega = \omega_1$).
3. Derive an approximate solution for $x_T(t)$ when $\omega \approx 0.1\omega_0$. Draw a sketch of $x_T(t)$ with x_T as the vertical axis and t as the horizontal axis.