2018
The Graduate School Entrance Examination
Physics
1:00 pm－3:00 pm

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Fill in your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of each answer sheet represent the problem number that you answer (P 1, P 2, P 3, P 4) and also the class of the course (master M, doctor D) that you are applying. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks per sheet.

6. You may use the blank sheets of the problem booklet for rough papers without detaching them.

7. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

I. Figure 1.1 shows a rocket flying upward in the vertical $+z$ direction. The rocket is propelled by expulsion of a gas in the opposite direction. The gas is expelled with constant velocity u relative to the rocket in the direction opposite to the advancing direction of the rocket. Except for the propulsive force, assume that all external forces such as air resistance and gravitation do not apply to the rocket. Answer the following questions.

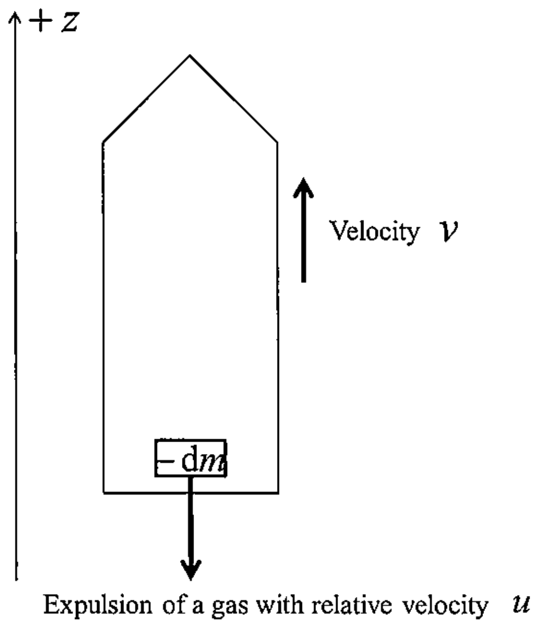

Figure 1.1

1. During a short interval $\mathrm{d}t$, from t to $t+\mathrm{d}t$, the rocket ejects some gas, and the velocity of the rocket increases from v to $v+\mathrm{d}v$. During this time, the mass of the rocket changes from m to $m+\mathrm{d}m$ ($\mathrm{d}m<0$). The momentum of the rocket at time t is equal to the sum of the momentum of the rocket at time $t+\mathrm{d}t$, and the momentum of the gas (with the mass of $-\mathrm{d}m$ (>0)) expelled during $\mathrm{d}t$. Under these conditions, find the relational expression between $\mathrm{d}v$, $\mathrm{d}m$, m and u. You may neglect the second-order terms of small values, such as $\mathrm{d}v\cdot\mathrm{d}m$.
2. The rocket continues to expel the gas for a long time, and the mass of the rocket decreases from m_i to m_f. Find the total increase in the velocity of the rocket.

II. Now assume that gravitational acceleration g applies to the rocket downward in the vertical $-z$ direction in Question I. Answer the following questions.

1. Find the time-rate-of-change in the velocity of the rocket v as a function of t on the basis of change in the momentum at short interval $\mathrm{d}t$. Here, the gas is expelled at a constant rate, such that m can be described by the following equation

$$m = m_i(1-kt), \quad (1)$$

where k is a positive constant.

2. The rocket continues to expel the gas for a long time, and then the velocity of the rocket increases from v_i to v_f as the mass of the rocket decreases from m_i to m_f. Find an expression for v_f.

III. Figure 1.2 shows the rocket flying in the upward vertical $+z$ direction. Assume that the rocket moves in the zx-plane. The axis of the rocket is tilted at an angle θ against the upward vertical $+z$ direction. Forces $L = K_L\theta$ and $D = K_D\theta$ are applied to the rocket along the horizontal ($+x$) and the vertical ($-z$) directions, respectively, at a position which is at distance ℓ_1 on the axis in front of the center of gravity G of the rocket. K_L and K_D are positive constants, respectively. θ is initially very small but non-zero. Additionally, the rocket engine is set at the bottom of the rocket on the axis (at a distance ℓ_2 from G), and provides a force of constant magnitude F to the terminal end of the rocket. The angle of this force can be changed to stabilize the rocket. The angle between F and the axis of the rocket is δ, as shown in Figure 1.2. Herein, the moment of inertia around the center of gravity G of the rocket along the axis perpendicular to the zx-plane can be expressed by I, which is assumed to be independent of time. Providing that θ and δ are very small, $\sin\theta$ and $\sin\delta$ can be approximated to $\sin\theta \cong \theta$ and $\sin\delta \cong \delta$, respectively, while $\cos\theta$ and $\cos\delta$ can be approximated to $\cos\theta \cong 1$ and $\cos\delta \cong 1$, respectively. Answer the followings.

1. Derive an equation of rotational motion for the rocket, and find a second-order differential equation of θ as a function of time t. Neglect terms of θ^2.

2. Show that $|\theta|$ increases with time when $\delta = 0$.
3. When δ is controlled such that $\delta = \alpha\theta$, find the condition on the constant α such that θ converges to zero as $t \to \infty$.
4. When δ is controlled such that $\delta = \alpha\theta + \beta(\mathrm{d}\theta / \mathrm{d}t)$, find the condition on the constants α and β such that θ converges to zero as $t \to \infty$.

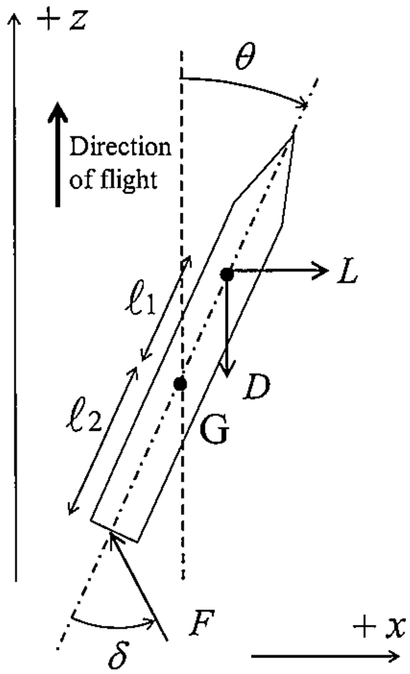

Figure 1.2

Problem 2

As shown in Figure 2.1, a parallel plate capacitor is placed in vacuum. The length and width of the capacitor are a and b, respectively. The gap between the two metal electrodes is d, which is much smaller than both a and b $(d << a, b)$. Let the permittivity of the vacuum be ε_0. A dielectric of permittivity $\varepsilon\ (> \varepsilon_0)$ is inserted in the gap. The length, width, and thickness of the dielectric are $a/2, b$, and d, respectively. The position of the left edge of the dielectric is labeled by x $(0 \le x < a)$, as shown in Figure 2.1. This dielectric can be moved in the range of $0 \le x < a$ without friction. When $0 \le x \le a/2$, the dielectric fits completely between the electrodes. On the other hand, when $a/2 < x < a$, the dielectric partly lies outside of the electrodes.

As shown in Figure 2.2, one can switch the connection between this capacitor and terminals A, B, or C for selecting one of three types of circuit. Answer the following questions. Neglect the capacitance, inductance, and resistance except for those shown in Figure 2.2.

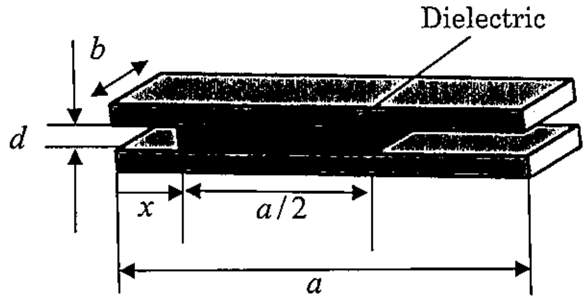

Figure 2.1

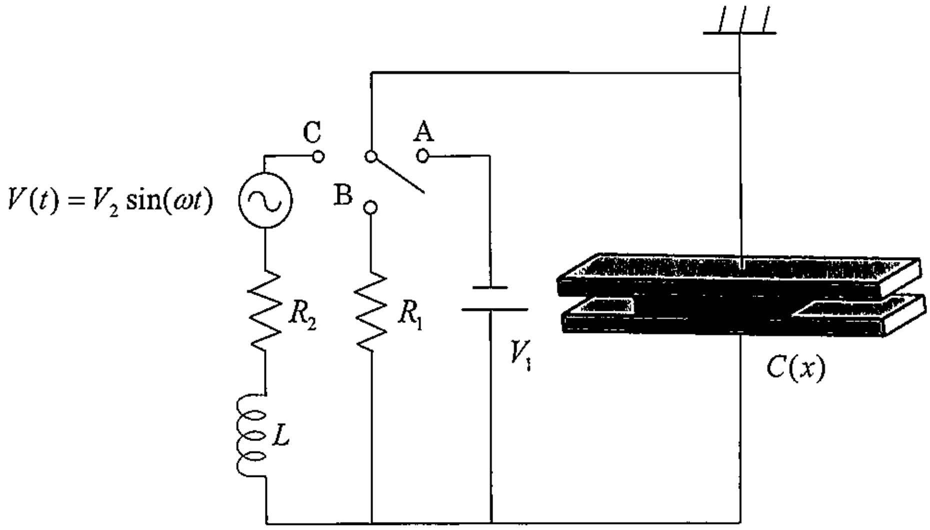

Figure 2.2

I. The switch is connected to terminal A and left for a long time. The constant DC voltage across the capacitor is V_1.

1. Derive the capacitance of the capacitor $C(x)$ as a function of x within the range of $0 \le x < a$. Use what you need from $x, \varepsilon_0, \varepsilon, a, b, d, V_1$.
2. Derive the energy stored in the capacitor $U(x)$ as a function of x within the range of $0 \le x < a$. Use what you need from $x, \varepsilon_0, \varepsilon, a, b, d, V_1$.
3. Derive the change of charge $\Delta Q(x)$ in the capacitor when the dielectric is slowly moved over a distance Δx within the range of $a/2 < x < a$. Use what you need from $\Delta x, x, \varepsilon_0, \varepsilon, a, b, d, V_1$.
4. Derive the force $F_1(x)$ acting on the dielectric along the horizontal direction as a function of x. Use what you need from $x, \varepsilon_0, \varepsilon, a, b, d, V_1$. Let the range of x be $0 \le x < a$. The direction of the force is defined as positive when the force acts on the dielectric such that the value of x increases (such that the dielectric moves towards the right in Figure 2.1).

II. The dielectric is set at $x = 3a/4$ and the switch is connected to terminal A. After a long time, the switch is switched from terminal A to terminal B. Let the time be $t = 0$ when the switch is switched. The resistance in the connected circuit is R_1. Derive the force acting on the dielectric $F_2(t)$ as a function of t. Use what you need from $t, x, \varepsilon_0, \varepsilon, a, b, d, V_1, R_1$. The direction of positive force is defined as in Question I.4. Let R_1 be large enough such that the magnetic field induced by the displacement current in the capacitor can be neglected.

III. The switch is connected to terminal C. The resistance in the connected circuit is R_2 and the inductance of the coil is L. The AC voltage of the electric power source is given by $V(t) = V_2 \sin(\omega t)$, where ω is the angular frequency.

1. The effective value of the power consumption, $P_2(x)$, in the resistor R_2 can be expressed as a function of x. Derive the condition of L for $P_2(x)$ to have a local maximum in the range of $0 \le x < a$. Use what you need from $\varepsilon_0, \varepsilon, a, b, d, V_2, R_2, \omega$.
2. Sketch a graph of the voltage waveform applied on the capacitor when $P_2(x)$ becomes maximum under the condition derived in Question III.1. Also sketch the voltage waveform of the electric power source on the same graph. Explain the physics behind the relation between the two waveforms.

Problem 3

I. Consider two solid objects, a and b, at temperatures T_1 and T_2, respectively, which have the same heat capacity C and are isolated from the outer environment. When object a is brought into contact with object b, the temperatures of these objects become the same, and the system reaches a thermal equilibrium state. Note that the volume of these objects does not change after the contact. Calculate the temperature T_f in the thermal equilibrium state and the change in entropy of this system, ΔS, after the objects are brought into contact. Prove that the entropy increases $(\Delta S > 0)$.

II. Consider an irreversible heat engine A and a reversible heat pump B, which operate between two heat reservoirs with constant temperatures as in Figure 3.1. The irreversible heat engine A absorbs heat Q_2^A from the high-temperature heat reservoir R_2, performs work W, and releases heat Q_1^A to the low-temperature heat reservoir R_1 in one cycle. The reversible heat pump B absorbs heat Q_1^B from the low-temperature heat reservoir R_1 through the work W generated by the irreversible heat engine A, and provides heat Q_2^B to the high-temperature heat reservoir R_2. Here, the efficiency of the irreversible heat engine and the reversible heat pump are denoted as $\eta_A (= W / Q_2^A)$ and $\eta_B (= W / Q_2^B)$, respectively. From the following relationships (a)-(c), choose all which do not satisfy the second law of thermodynamics, and explain why.

(a) $\eta_A < \eta_B$, (b) $\eta_A = \eta_B$, (c) $\eta_A > \eta_B$.

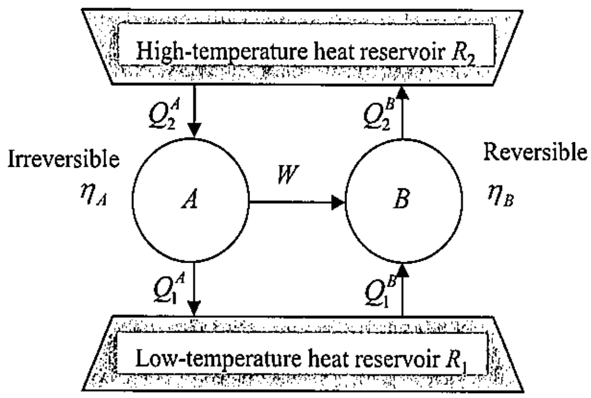

Figure 3.1

III. Consider a closed vessel filled with a photon gas at temperature T in an isolated environment. In a thermal equilibrium state, the pressure p of the photon gas, which is independent of volume V, is described with the following equation

$$p = \frac{1}{3} a T^4 , \tag{1}$$

where a is a positive constant. Answer the following questions.

1. Explain why this photon gas does not have a heat capacity at constant pressure.
2. When the volume of the vessel V is quasi-statically changed at a constant temperature, according to the first law of thermodynamics, Equation (2) is true, where U is the internal energy of the photon gas and S is its entropy. The change in the internal energy is described with Equation (3). Prove Equation (3) using the first law of thermodynamics. You may use the Maxwell relation as in Equation (4).

$$dU = TdS - pdV . \tag{2}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p . \tag{3}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V . \tag{4}$$

3. Calculate the internal energy $U(T,V)$ and entropy $S(T,V)$ of the photon gas in the thermal equilibrium state. Here, the internal energy and the entropy are both zero at absolute zero $(T = 0)$.
4. Figure 3.2 shows a Carnot cycle in which the working medium is a photon gas. This cycle consists of four processes: isothermal expansion (A→B), adiabatic expansion (B→C), isothermal compression (C→D), and adiabatic compression (D→A). The amount of heat that the system absorbs in the isothermal expansion process at temperature T_2 is denoted as Q_2, and the amount of heat that the

system releases in the isothermal compression process at temperature $T_1(<T_2)$ is denoted as Q_1. Calculate Q_2 and Q_1. Here, the quantity of heat is defined as positive when the system absorbs heat.

5. Show that the Clausius equality (5) is true in this case.

$$\frac{Q_2}{T_2}+\frac{Q_1}{T_1}=0 \qquad (5)$$

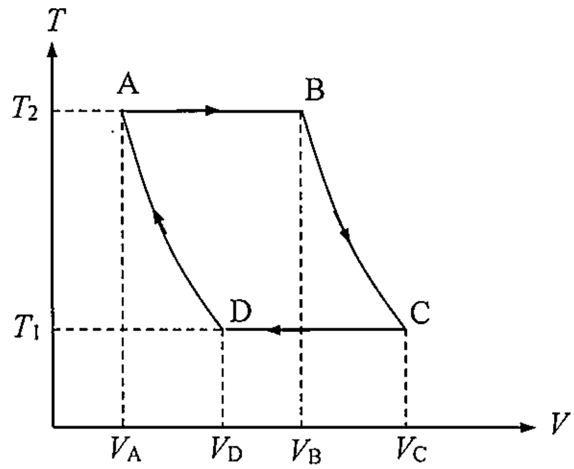

Figure 3.2

Problem 4

Answer the following questions about the wave nature of light and particles and also relativistic effects.

I. As shown in Figure 4.1, light passing through a single slit and then double slits propagates towards a screen. Interference fringes are generated on the screen. Answer the following questions. Here, the widths of all slits are much smaller than the wavelength of the light.

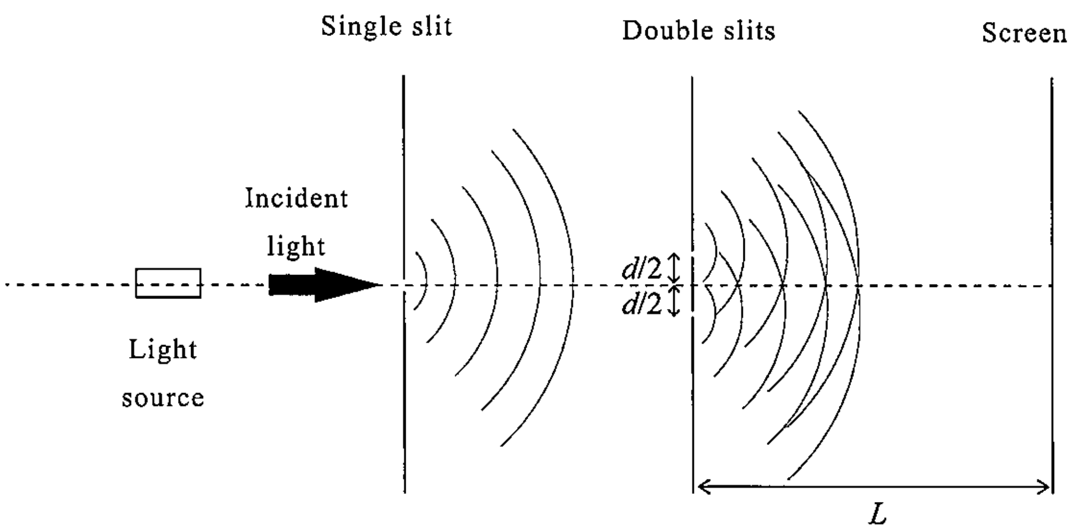

Figure 4.1

1. The incident light source produces monochromatic light with a visible wavelength of λ. The separation of the double slits is d, as shown in Figure 4.1. The distance between the double slits and the screen is L. Under these conditions, express the interval x of the interference fringes on the screen, using λ, d, and L. Here, you can assume that L is much larger than d and λ.
2. Next, the incident light source is changed from the monochromatic one to a white one. Explain the differences between the pattern on the screen in the case of monochromatic light and white light. You may draw figures if needed.

II. Instead of light, now consider that an electron beam passes through the double slits and propagates towards the screen shown in Figure 4.1. In quantum mechanics, electrons have wave-particle duality. Therefore, as with light, interference fringes of electrons appear on the screen. The wavelength λ of electrons is given by $\lambda = h/p$, where h is Planck's constant and p is the momentum of the electrons. Answer the following questions.

1. The mass and the charge of an electron are m and e, respectively. Express the wavelength λ of the electron accelerated by an electric potential difference V in vacuum, using m, e, h, and V. Here, you can ignore any relativistic effects.
2. Calculate the value of the electric potential difference V necessary for generating an electron with a wavelength of 2.0 Å, using the equation that you derived in Question II.1. Use the following values for your calculation.

Planck's constant $h = 6.6\times10^{-34}$ [J s]
Charge of electron $e = 1.6\times10^{-19}$ [C]
Mass of electron $m = 9.1\times10^{-31}$ [kg]

III. As the velocity of accelerated electrons approaches the velocity of light, relativistic effects cannot be ignored. Answer the following questions. Here, Planck's constant and charge of electron is h and e, respectively.

1. (x, y, z) and (x', y', z') are defined as coordinate systems in two inertial frames, S and S', respectively. t and t' are times in S and S', respectively. Now, the two coordinate systems move in parallel. At times $t = t' = 0$, the coordinate origin and clock of S' exactly coincide with those of S. S' is moving at a constant velocity v in the positive direction along the x-axis when seen from S. The coordinate transformations between S and S' are defined with the following equations

$$x' = \alpha(x - vt), \quad y' = y, \quad z' = z. \tag{1}$$

$$x = \alpha(x' + vt'), \quad y = y', \quad z = z'. \tag{2}$$

A pulse of light is emitted from the origin in the positive direction along the x-axis at times $t = t' = 0$. The speed of light c is invariant in both inertial frames. By considering the positions of the light at time t in S and at time t' in S', express the positive coefficient α in terms of v and c.

2. Considering relativistic effects, the momentum p of the electron with uniform motion at a velocity v is expressed as

$$p = \alpha m_0 v, \tag{3}$$

where the coefficient α is that which was obtained in Question III.1, and m_0 is the static mass of the electron. The total energy E of this electron is expressed as

$$E^2 = m_0^2 c^4 + p^2 c^2. \tag{4}$$

Here, the rest energy is defined as $m_0 c^2$.

Consider an electron with a constant velocity after being accelerated by an electric potential difference V in vacuum. Express the wavelength λ and the velocity v of this electron, using all or some of c, m_0, e, V, and h. Here, you must take relativistic effects into account.